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ФИЗИКА НЕЙТРИНО. ЧАСТЬ 2

СТУДЕНИКИН
АЛЕКСАНДР ИВАНОВИЧ

ФИЗФАК МГУ

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Lecture 1. Electromagnetic properties of neutrino

Magnetic moment of neutrino

We know that the mass of neutrino is not zero. From KATRIN experiment we have the following limit on the effective neutrino mass:

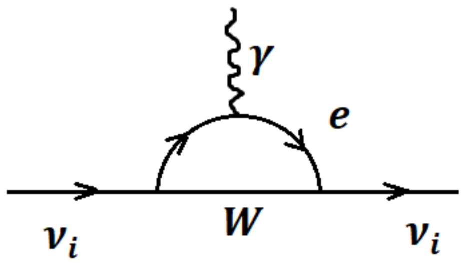
$$m_{eff} < 0.8\ eV \quad (1.1)$$

If we use the Standard Model easiest generalization which demands that there is a right handed neutrino ν_R and the mass of neutrino is not zero $m_\nu \neq 0$ it is immediately follows that the magnetic moment of neutrino is not zero $\mu_\nu \neq 0$. This phenomenon was discussed many years ago even before neutrino was experimentally observed in 1956.

Let's illustrate how this non-zero magnetic moment appears in the Standard Model generalization. There is a part in SM Lagrangian which corresponds to the coupling of neutrino with electron and W-boson:

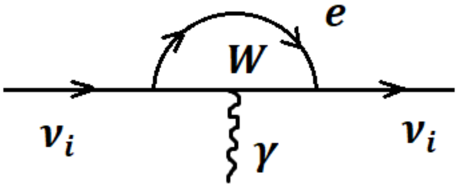
$$\mathcal{L} \sim \bar{\nu}\gamma^\mu(1-\gamma_5)eW_\mu\left(-\frac{g}{2\sqrt{2}}\right) \quad (1.2)$$

In (1.2) we consider flavor neutrinos which are superpositions of mass states. For the mass states of neutrino we can write the following diagrams:



Pic.1.1. Neutrino interaction diagram.

If we recall the quantum electrodynamics for electron such diagram (pic.1.1) contributes to the anomalous magnetic moment of electron in addition to the normal magnetic moment determined by non-zero electric charge. In case of neutrinos up to now we don't discuss the possibility of millicharged neutrinos. But The Particle Data Group collaboration once in two years publishes the sets of properties including constraints on the millicharge of neutrino. I am proud to say that our scientific group contributed a lot to these studies. We analyzed the data for scattering of neutrino from reactor on electrons in GEMMA experiment, included the possibility that the charge of neutrino is not zero and obtained the best limit on the millicharge of neutrino. We can also write the following diagram:



Pic.1.2. Neutrino interaction diagram.

Calculating the diagrams (pic.1.1-1.2) Schrock and Fujikawa in 1982 obtained the following result for the anomalous magnetic moment of Dirac neutrino:

$$\mu_{ii}^D = \frac{3eG_F m_i}{8r_2\pi^2} \approx 3{,}2 \cdot 10^{-19} \left(\frac{m_i}{1eV}\right)\mu_B, \tag{1.3}$$

$$\mu_B = \frac{e}{2m_e} \tag{1.4}$$

We should notice that the magnetic moment of neutrino is proportional to its mass. According to the fact that the mass of neutrino is not zero we immediately get that neutrino will participate in electromagnetic interactions. There is a shift of energy of neutrino that is proportional to its magnetic moment and external magnetic field:

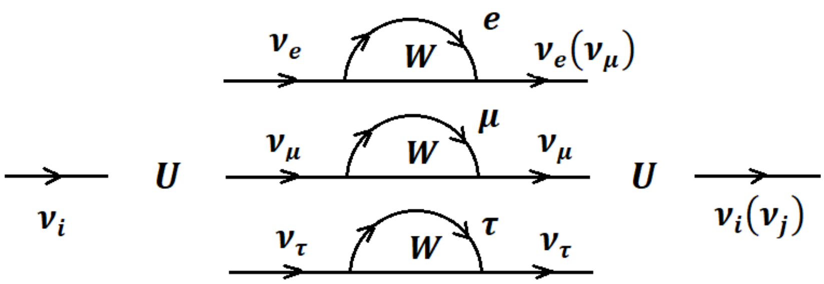
$$\Delta W_\nu \sim \mu B \tag{1.5}$$

If magnetic field is strong, for example, in astrophysics where

$$B_{ast.} \sim B_\theta = \frac{m_e^2}{e} = 4{,}41 \cdot 10^{13}\ G, \tag{1.6}$$

even with very small magnetic moment of neutrino that predicted by the easiest generalization of the Standard Model we can expect that the value ΔW_ν will be significant. In case of very small magnetic fields, for example, interstellar and intergalactic fields $\sim 10^{-9} \div 10^{-6}\ G$ you need to wait a lot of time to see a reasonable result.

Using the mixing matrix U we can write the mass state of neutrino as a superposition of flavor neutrinos for which we can already use the Standard Model Lagrangian (1.2). So we get the following diagram:



Pic.1.3. Neutrino interaction diagram.

In more general case this interaction may change the type of neutrino. Therefore we should distinguish diagonal magnetic moments μ_{ii}^{D} and off diagonal or transition magnetic moments $\mu_{i\neq j}^{D}$.

Experimental constraints on neutrino magnetic moment

In terrestrial experiments physicists observe the scattering of neutrinos on electrons including not only Standard Model weak interaction but also an interaction of the neutrino magnetic moment. In 2012 in GEMMA (JINR-Dubna and ITEP-Kurchatov) experiment they only saw the background. It means that the magnetic moment couldn't exceed the background, therefore, there is the limit on the neutrino magnetic moment:

$$\mu_{ii}^{D} \leq 2{,}9 \cdot 10^{-11} \mu_B \tag{1.7}$$

If $m_i \neq 0$ the Standard Model predicts that $\mu_\nu \sim 10^{-19} \mu_B$.
The Borexino Collaboration (Gran Sasso, Italy) investigated the solar neutrinos and obtained following the limit on neutrino magnetic moment:

$$\mu \leq 2{,}8 \cdot 10^{-11} \mu_B \tag{1.8}$$

More severe bound could be obtained from astrophysical experiments:

$$\mu_{\text{ast.}} \leq 10^{-13} \mu_B \tag{1.9}$$

The left handed neutrino propagating in astrophysical environment with quite strong magnetic field due to interaction of non-zero magnetic moment can transfer to the right handed neutrino:

$$\nu_L \xrightarrow{B} \nu_R, \tag{1.10}$$

$\nu_L -$ left handed active neutrino, $\nu_R -$ right handed sterile neutrino. We can expect that sterile neutrinos are invisible for the detector. It means that the flux of left handed active neutrinos will be less than expected. On the other hand this transition from left handed to right handed neutrinos disturbs the process of evolution of the star. If we consider a very compact object, for example, supernova, most of the energy is evaporated due to neutrino fluxes. If inside the star very strong magnetic field exists than left handed active neutrinos will convert to the right handed sterile neutrinos and after that immediately escape without any interaction with the body of the star. It means that the flux of these neutrinos is produced more dipper in the central part of the star and, therefore, the energy of these neutrinos can be higher than the energy of left handed neutrinos that somehow propagated inside the star and then had chance to escape. It means that the energy spectrum of such astrophysical neutrinos will be disturbed by the interaction of the magnetic moment with the magnetic field inside the star.

I would like to mention that in the National Centre of Physics and Mathematics in Sarov there is a study of the coherent neutrino atom scattering with the goal to put severe bound on the neutrino magnetic moment. In 2019 together with my collaborators we published a paper where we calculated the scattering of neutrino on a Helium target accounting the scattering on nuclei and electrons. We proposed that this scheme of experiment will be very sensitive to the magnetic moment of neutrino. We predicted that the sensitivity will be

$$I_{\mu} \sim 10^{-13} \mu_B \tag{1.11}$$

Chiral and helicity neutrino states

We can decompose neutrino wave function to chiral states. Chiral states ν_L and ν_R actually enter the Standard Model Lagrangian. For left handed and right handed neutrino states we can write:

$$\psi_L = \frac{1-\gamma_5}{2}\psi, \tag{1.12}$$

$$\psi_R = \frac{1+\gamma_5}{2}\psi \tag{1.13}$$

Total neutrino wave function is a superposition of two chiral states:

$$\psi = P_L\psi + P_R\psi = \psi_L + \psi_R, \tag{1.14}$$

$P_L, P_R -$ chiral projector operators.

There is another possibility to decompose neutrino wave function. Helicity is a property that determined by the orientation of the polarization over the momentum of neutrino. We can introduce the helicity projector operators:

$$P_{\pm} = \frac{1}{2}\left(1 \mp \frac{\vec{\Sigma}\vec{p}}{|\vec{p}|}\right), \tag{1.15}$$

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \tag{1.16}$$

Therefore, the helicity neutrino states:

$$\psi_{\pm} = \frac{1}{2}\left(1 \mp \frac{\vec{\Sigma}\vec{p}}{|\vec{p}|}\right)\psi \tag{1.17}$$

In case when $m_\nu = 0$ the helicity and chiral states coincide with each other:

$$\psi_L \approx \psi_-, \tag{1.18}$$

$$\psi_R \approx \psi_+ \tag{1.19}$$

For free propagating neutrinos helicity is conserved. It means that the $P_{\pm}$ operators commute with the Hamiltonian but γ_5 not.

Neutrino mixing and oscillation in electromagnetic field

We shall deal with a new phenomenon of mixing between different helicity and chiral states of neutrinos. We will consider the case of magnetic field. So we have left handed and right handed neutrinos ν_L, ν_R with non-zero mass $m \neq 0$ and therefore non-zero magnetic moment $\mu \neq 0$ that will interact with magnetic field B. The Lagrangian will look like

$$\mathcal{L} = \mu\bar{\nu}\sigma_{\lambda\rho}F^{\lambda\rho}\nu' \tag{1.20}$$

We include the possibility that neutrino in initial and final states don't equal to each other $\nu \neq \nu'$. It means that we may deal with the transition magnetic moment. Weak interactions are introduced in flavor states but electromagnetic interactions better be introduced in mass states. It is possible to show that this coupling (1.20) is a mix of different helicity or chiral states:

$$\nu = \nu_L + \nu_R \Rightarrow \tag{1.21}$$

$$\mathcal{L} \sim \bar{\nu}_L \sigma_{\lambda\rho} F^{\lambda\rho} \nu'_R + \bar{\nu}_R \sigma_{\lambda\rho} F^{\lambda\rho} \nu_L \tag{1.22}$$

So we introduced the mixing between the left and right neutrinos. We can see that this phenomenon is the same with the mixing for the flavor or mass states due to mixing matrix. Therefore there are also oscillations between the left and right neutrinos.

The probability of oscillations left and right neutrinos in magnetic field

We will consider a constant magnetic field but with the twisting of vector's direction. This model is often used to describe the structure of magnetic field in some astrophysical objects, for example, the Sun. We can decompose magnetic field in two components in respect to direction of neutrino propagation z:

$$\vec{B} = \vec{B}_z + i\vec{B}_y \tag{1.23}$$

Twisting magnetic field is described as:

$$B = |\vec{B}_\perp| e^{i\phi(t)} \tag{1.24}$$

It is a constant field that rotates in space while neutrino propagates through it.

We consider two different flavor neutrinos with two different chiral states. Let's write the evolution equation for neutrino:

$$i\frac{d}{dt}\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix} = H\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix}, \tag{1.25}$$

$$H = \begin{pmatrix} E_L & \mu_{e\mu} B e^{-i\phi(t)} \\ \mu_{e\mu} B e^{-i\phi(t)} & E_R \end{pmatrix} = (\dots)I + \widetilde{H} \tag{1.26}$$

As soon we are interested in the probability of oscillations we can skip the part that is proportional to the unit matrix. We consider electrically neutral matter composed of electrons, neutrons and protons ($n_e = n_p$). So only electron components will contribute to the evolution and the oscillation probability.

$$\widetilde{H} = \begin{pmatrix} -\frac{\Delta m^2}{4E^2}\cos 2\theta + \frac{V_{\nu_e}}{2} & \mu_{e\mu} B e^{-i\phi(t)} \\ \mu_{e\mu} B e^{i\phi(t)} & \frac{\Delta m^2}{4E^2} - \frac{V_{\nu_e}}{2} \end{pmatrix} \tag{1.27}$$

We see that the Hamiltonian (1.27) includes the mixing between electron and muon neutrino and also the additional mixing between left and right neutrinos of different spices.

It is possible to find the solution for evolution equation. We can introduce the unitary transformation for neutrino spices using diagonal operator. So the probability of oscillations in the initial and in the final basis will be the same. New basis will be

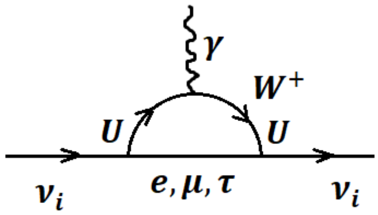
$$\nu = U\nu', U = \begin{pmatrix} e^{-\frac{i\phi}{2}} & 0 \\ 0 & e^{\frac{i\phi}{2}} \end{pmatrix} \tag{1.28}$$

Lecture 2. Neutrino evolution in constant magnetic field

Introduction

We have started the investigation of neutrino mixing and oscillations in the presence of external electromagnetic field. This is a very important issue related to neutrino electromagnetic properties. We consider the case that the mass of neutrino is not zero $m_\nu \neq 0$ which is beyond the Standard Model. The Standard Model supposed that the mass of neutrino is zero but it is proven experimentally that if the mass of neutrino is zero there are no mixing and oscillations and no chance to explain the observed neutrino fluxes from the Sun and in atmosphere. In fact there are three different mass states of neutrino $\begin{matrix}\nu_1\\ \nu_2\\ \nu_3\end{matrix}$ and three different flavor states $\begin{matrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{matrix}$ which are superpositions of these mass states. From experimental results we know that at least two of three masses of mass states m_1, m_2, m_3 are non-zero and there are no two mass states with equal masses.

So these experimental results force us to go beyond the Standard Model. If we just consider the easiest generalization of the Standard Model using the same Lagrangian and adding a single right-handed neutrino state to have the mass term in the Lagrangian we immediately get that all interactions that exist in the Standard Model should be described within new theory beyond the Standard model. If so there is contribution to the neutrino magnetic moment $\mu_\nu \neq 0$ from this kind of diagrams:



Pic. 2.1. One loop diagram.

So we express the mass state as a superposition of flavor states, and each of these states should interact with W^+-boson. We get that the neutrino magnetic moment in the mass basis is proportional to the mass of the mass state $\mu_i \sim m_i$.

Now we should deal with the equation that describes neutrino motion in an external electromagnetic field accounting for the fact that neutrino has non-zero magnetic moment. We are going to describe more general case when we include not only electromagnetic fields but also matter which can move in respect to direction of neutrino propagation. We discussed many interesting effects related also to the interaction of neutrinos with the moving matter. I have mentioned that in the presence of transversal magnetic field a new type of mixing and oscillations appear between left and right-handed neutrinos. But about 20 years ago in our paper we have discovered that the mixing of spin states and corresponding oscillations can be produced without any electromagnetic properties, magnetic moment and presence of external electromagnetic or magnetic field but due to weak interactions of neutrinos with moving matter if the transversal component of the flux of matter exist. So we have shown that spin oscillations can be produced in case when neutrino propagates inside moving matter and when there is a transversal current of matter acting on neutrinos and producing spin flip. This phenomenon can have important consequences in astrophysics.

Effective Dirac-Pauli equation in an external electromagnetic field

We consider neutrino as electrically neutral particle. We can write Dirac equation in external electromagnetic field:

$$\left(i\gamma_\mu \partial^\mu - m_i + \frac{1}{2}\mu_i \sigma_{\mu\nu} F^{\mu\nu}\right) \nu_i(x) = 0, \tag{2.1}$$

where $m_i -$ mass of the mass state, $\mu_i -$ magnetic moment of the mass state of neutrino, $\nu_i(x) -$ wave function of the flavor state in coordinate space. By mixing we can determine electromagnetic characteristics in the flavor basis. Because Lagrangian weak interaction is determined in terms of flavor neutrinos and most of experiments deal with flavor neutrinos. Tensor $\sigma_{\mu\nu}$ is a combination of Dirac matrices:

$$\sigma_{\mu\nu} = \frac{i}{2}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) \tag{2.2}$$

Using Fourier transform of (2.1) we get

$$\left(\gamma_\mu p^\mu - m_i + \frac{\mu_i}{2}\sigma_{\mu\nu} F^{\mu\nu}\right) \nu_i(p) = 0, \tag{2.3}$$

where $\nu_i(p) -$ wave function of the flavor state in momentum space.

We are interested in case of constant magnetic field $F^{\mu\nu} \rightarrow \vec{B}, \vec{B} = (B_1, B_2, B_3)$. In this case we have the following expression for the tensor of electromagnetic fields:

$$F^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -B_3 & B_2 \\ 0 & B_3 & 0 & -B_1 \\ 0 & -B_2 & B_1 & 0 \end{pmatrix} \tag{2.4}$$

We can continue our calculations in (2.3):

$$\frac{\mu_i}{2}\sigma_{\mu\nu} F^{\mu\nu} = i\mu_i(\sigma_{21}B_3 + \sigma_{13}B_2 + \sigma_{32}B_1) = -\mu_i \vec{\Sigma}\vec{B}, \tag{2.5}$$

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \tag{2.6}$$

Recall the expression for the Pauli matrices:

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{2.7}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \tag{2.8}$$

$$\sigma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{2.9}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2.10}$$

Neutrino evolution in constant magnetic field

We are going to derive an effective Hamiltonian for the mass states of neutrino that will describe neutrino propagation, in vacuum first, accounting for the neutrino magnetic moment interaction with a constant magnetic field. Magnetic field has two components in respect to neutrino propagation $\vec{p}_\nu$:

$$\vec{B} = \vec{B}_{||} + \vec{B}_{\perp} \tag{2.11}$$

We would like to find an effective equation of evolution of the mass states of neutrino in vacuum accounting for the neutrino magnetic moment interaction with a constant magnetic field and, therefore, an effective Hamiltonian for the mass states in magnetic field. In case when magnetic field is zero $\vec{B} = 0$:

$$H_{eff} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}, \tag{2.12}$$

$$E_\alpha = \sqrt{m_i^2 + |\vec{p}|^2}, \alpha = 1,2 \tag{2.13}$$

In case of interaction of neutrino with magnetic field due to non-zero magnetic moment an additional term will appear:

$$\widetilde{H}_{eff} = H_{eff} + H_{\mu B} \tag{2.14}$$

We would like to find this shift $H_{\mu B}$ on effective evolution Hamiltonian for the mass states of neutrino. From Lagrangian $\mathcal{L}_B$ of neutrino interaction with the magnetic field we can obtain effective Hamiltonian H_B. Lagrangian itself comes from the Dirac-Pauli equation for the mass states of neutrino ν_i with non-zero magnetic moment $\mu_i \neq 0$ in magnetic field B. Finally, the Hamiltonian that corresponds to the Lagrangian $\mathcal{L}_B$ is

$$H_B^{\mathcal{L}} = -\frac{1}{2}\mu\bar{\nu}\sigma_{\mu\nu}\nu F^{\mu\nu} + h.c. \tag{2.15}$$

For the further needs I would like to introduce more complicated electromagnetic property. We suppose that not only this electromagnetic interaction of neutrinos with an external magnetic field is described but also the mass state of neutrino is changing:

$$H_B^{\mathcal{L}} = -\frac{1}{2}\mu_{12}\bar{\nu}_1\sigma_{\mu\nu}\nu_2 F^{\mu\nu} + h.c., \tag{2.16}$$

where $\mu_{12} -$ transition (off diagonal) magnetic moment. So, μ_{ii} is magnetic moment of ν_i, $\mu_{i\neq j} -$ transition magnetic moment of neutrino ν. The contribution to the effective Hamiltonian of neutrino evolution that enters Schrodinger evolution equation for our neutrino system:

$$\langle \nu_1 | H_B^{\mathcal{L}} | \nu_2 \rangle = ?$$

We use an exact neutrino state ν_i that corresponds to the mass, the momentum and to the energy as a solution of the vacuum Dirac equation:

$$\nu_1(m_1, p_1, E_1) = C_1\sqrt{\frac{E_1 + m_1}{2E_1}} \begin{pmatrix} u_1 \\ \frac{\vec{\sigma}\vec{p}_1}{E_1 + m_1} u_1 \end{pmatrix} exp\{i(-E_1 t + \vec{p}_1\vec{x})\}, \tag{2.17}$$

$$\nu_2(m_2, p_2, E_2) = C_2\sqrt{\frac{E_2+m_2}{2E_2}}\begin{pmatrix} u_2 \\ \frac{\vec{\sigma}\vec{p}_2}{E_2+m_2}u_2 \end{pmatrix} exp\{i(-E_2t+\vec{p}_2\vec{x})\} \quad (2.18)$$

Calculating the average (2.17) we just get a typical energy related to transition between different neutrino states:

$$\langle\nu_1|H_B^{\mathcal{L}}|\nu_2\rangle = -\frac{1}{2}\int d^4x\nu_1^+\gamma_0\begin{pmatrix} \vec{\sigma}\vec{B} & 0 \\ 0 & \vec{\sigma}\vec{B} \end{pmatrix}\nu_2 \quad (2.19)$$

To describe the spin states of neutrino we agree that $u_\alpha, \alpha = 1{,}2$ is an additional characteristic that describes the spin state:

$$u_{\alpha,s=\frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad (2.20)$$

$$u_{\alpha,s=-\frac{1}{2}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.21)$$

It means that we deal with two mass states of neutrino each in two possible spin states. In general our evolution Hamiltonian will be 4×4 matrix. So

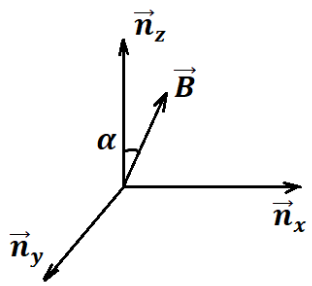
$$\frac{1}{2}\mu_{12}\int d^4x\vec{B}\left(u_1^+, \frac{\vec{\sigma}\vec{p}_1}{E_1+m_1}u_1\right)\begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}\begin{pmatrix} u_2 \\ \frac{\vec{\sigma}\vec{p}_2}{E_2+m_2}u_2 \end{pmatrix} exp\{i(-\Delta Et + \Delta px)\}\frac{\sqrt{(E_1+m_1)(E_2+m_2)}}{2\sqrt{E_1E_2}}, \quad (2.22)$$

$$\Delta E = E_2 - E_1, \quad (2.23)$$

$$\Delta p = p_2 - p_1 \quad (2.24)$$

The term $exp\{i(-\Delta Et + \Delta px)\}$ can be cancelled out uh due to renormalization. Without the loss of generality we suppose that both neutrino states move along axis z or the longitudinal component of magnetic field:

$$\vec{p}_1, \vec{p}_2 \parallel \vec{n}_z(\sim\vec{B}_{\parallel}) \quad (2.25)$$



Pic. 2.2. The vector of magnetic field.

Let's continue, we should calculate the following term:

$$u_1^+(\vec{B}\vec{\sigma})u_2 - \frac{u_1^+(\vec{\sigma}\vec{p}_1)(\vec{\sigma}\vec{B})(\vec{\sigma}\vec{p}_2)u_2}{(E_1+m_1)(E_2+m_2)}$$

$$= u_1^+ \left\{ (\vec{\sigma}\vec{B}_{\parallel}) \left(1 - \frac{\vec{p}_1\vec{p}_2}{(E_1 + m_1)(E_2 + m_2)} \right) + (\vec{\sigma}\vec{B}_{\perp}) \left(1 + \frac{\vec{p}_1\vec{p}_2}{(E_1 + m_1)(E_2 + m_2)} \right) \right\} u_2, \quad (2.26)$$

where we used the expression:

$$(\vec{\sigma}\vec{a})(\vec{\sigma}\vec{b}) = (\vec{a}\vec{b}) + i(\vec{\sigma}[\vec{a}\vec{b}]) \quad (2.27)$$

We should consider four combinations:

1) $s_+ s_+$:

$$u^+_{1,s=\frac{1}{2}}(\vec{\sigma}\vec{B})u_{2,s=\frac{1}{2}} = (1,0)\vec{\sigma}\begin{pmatrix}1\\0\end{pmatrix}(\vec{B}_{\parallel} + \vec{B}_{\perp}) = B\cos\alpha, \quad (2.28)$$

where we agreed that

$$|\vec{B}_{\parallel}| = B\cos\alpha, \quad (2.29)$$

$$|\vec{B}_{\perp}| = B\sin\alpha \quad (2.30)$$

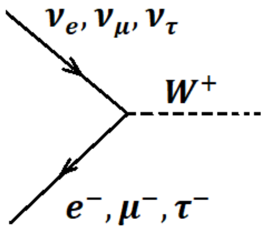
2) $s_+ s_-$
3) $s_- s_+$
4) $s_- s_-$

Lecture 3. Electromagnetic properties of neutrino (CONT'D)

Introduction. Neutrino properties

Our lectures are devoted to the study of neutrino electromagnetic properties. This is a key issue of the research that has been conducted for three decades by our research group. The widest review on the subject has been published in 2015 by my colleague and friend Carlo Ginti and me and it's called "Electromagnetic properties of neutrino: a window to a new physics". It was published in the "Reviews of Modern Physics, 87(2015),p.531-591".

Before continue our calculations that we have performed last time I would like to recall the present fundamental knowledge about the properties of neutrinos. Firstly, we know that the mass of neutrino is not zero $m_i \neq 0, i = 1,2,3$. Secondly, there are mixing and oscillations between neutrinos. There are flavor neutrinos $\nu^f, f = e, \mu, \tau$ that determined by the Lagrangian of interaction of the electro-weak theory. The Lagrangian of the Standard Model has the following type of interaction:



Pic. 3.1. Neutrino interactions.

According to the idea that was first formulated by Bruno Pontecorvo in 1957 there are a mixing and oscillations between different types of neutrino. At that time there was no notion about different types of neutrino. Muon neutrino was discovered in 1962. Pontecorvo spoke about mixing and oscillations between known at that time two types of neutrino that were recognized as neutrino ν and antineutrino $\bar{\nu}$ without identification of the flavor. Pontecorvo made a statement that if the mass of neutrino is not zero $m_\nu \neq 0$ then neutrino should be considered as a mixture or superposition of different types neutrino ν and $\bar{\nu}$. The second statement he made by words without any formulas was that on a certain distance from a reactor (reactor was considered as a source of neutrinos) the flux of neutrino will be composed of equal amount of neutrinos and antineutrinos. In 1962 the experimental discovery of muon neutrino was made by Japanese scientists Sakata, Maki and Nakagawa. (Tau neutrino was discovered only in 2000.) Sakata, Maki and Nakagawa generalized the idea of Pontecorvo and claimed that there is mixing not between neutrino and antineutrino but between two different types of neutrinos ν_e and ν_μ. They also introduced the mixing matrix $U = U(\theta)$ that can be parameterized by the mixing angle. But Sakata, Maki and Nakagawa have failed to go further and didn't follow the second statement Pontecorvo made in 1957 about evolution of neutrino flux in space and time. Only in 1969 Pontecorvo and Gribov considered the dynamic of these mixing neutrinos and first derived the oscillation probability.

Neutrino electromagnetic properties

Initially in 1930 W. Pauli proposed the existence of neutrino and called it neutron. Only after the real neutron was discovered by Thompson in 1932 Fermi renamed the neutron of Pauli to neutrino. When Pauli introduced neutrino he proposed that this particle is a fermion $s=\frac{1}{2}$, it has zero mass $m_\nu = 0$ and zero charge $q_\nu = 0$ but probably non-zero magnetic moment $\mu_\nu \neq 0$.

Let's consider the present status of these properties of neutrino. We know for sure that the mass of neutrino is not zero $m_\nu \neq 0$, it comes from two sets of experiment observing the fluxes of solar $\nu_\odot$ and atmospheric neutrinos ν_{atm}. The Standard Model fails to describe the experimental fluxes of solar and atmospheric neutrinos. The only explanation that can be found is the existence of mixing and oscillations between different types of neutrino. We know that there are three mass states of neutrino, at least two of three masses of mass states m_1, m_2, m_3 are non-zero and there are no two mass states with equal masses:

$$m_i \neq 0 \neq m_j, m_i \neq m_j \tag{3.1}$$

It follows from the fact that the oscillation probability that are used to provide the solution for solar neutrinos depends on the mass square difference

$$\Delta m_{12}^2 = m_2^2 - m_1^2 \neq 0 \tag{3.2}$$

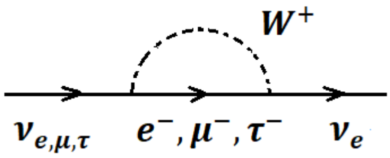
For the atmospheric neutrinos another scale of mass square difference should be needed:

$$\Delta m_{13}^2 = m_3^2 - m_2^2 \neq 0 \tag{3.3}$$

There are direct experimental attempts to measure the absolute value of these masses. The latest experiment was conducted by KATRIN collaboration in 2019 and delivered the best upper bound on the effective mass of neutrino:

$$m_{eff} < 0.8\ eV \tag{3.4}$$

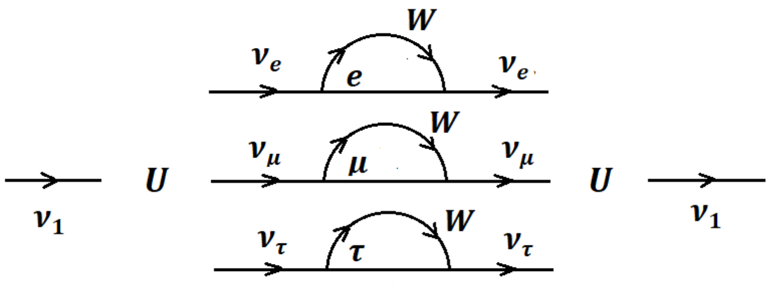
We know that the interactions of flavor neutrinos are described by the Lagrangian of the Standard Model. From all these definitions and properties that were mentioned above it follow that flavor neutrinos have no mass. If a flavor neutrino is a superposition of different mass states we can't introduce in a normal way the mass for the flavor neutrino and, therefore, we can't write the Dirac equation of motion. The Standard Model predicts the following interaction for flavor neutrino:



Pic. 3.2. Flavor neutrino interaction.

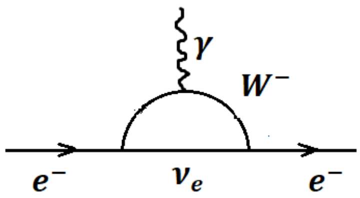
If you remember these kinds of diagrams due to such radiation loop can produce interaction with photons. According to the Standard Model the electric charge of flavor neutrino is zero $q_{\nu_e} = 0$ and the diagram on pic.3.2 gives a contribution to the anomalous magnetic moment of neutrino. This diagram can be written using the mass states of neutrino (pic. 3.3). Because flavor neutrinos as we know are superpositions of neutrino mass states we can decompose them using mixing matrix U:

$$\nu_f = \sum_{i=1,2,3} U_{if}\nu_i\,, f = e, \mu, \tau \tag{3.5}$$



Pic. 3.3. Flavor neutrino interaction.

We can write the similar diagram (pic. 3.2) for an electron (pic. 3.4) that will give contribution to the anomalous magnetic moment of electron $\Delta\mu_e$.



Pic. 3.4. Interaction diagram for an electron.

If $q_{\nu_e} = 0$ the diagram on pic.3.2 gives a contribution to the anomalous magnetic moment of neutrino. For the case of the Dirac neutrino ν^D the contribution to the magnetic moment proportional to the mass of the particle:

$$\mu_{ii}^D = \frac{3eG_F m_i}{8r_2\pi^2} \approx 3{,}2 \cdot 10^{-19}\left(\frac{m_i}{1eV}\right)\mu_B, \tag{3.6}$$

$$\mu_B = \frac{e}{2m_e} \tag{3.7}$$

We have three different magnetic moments $\mu_{11}, \mu_{22}, \mu_{33}$. From (3.1) it follows that at least two magnetic moments are not zero and not equal to each other.

However there are different theoretical models that provide quite different prediction for the neutrino magnetic moment. For instance, in the theory of super symmetry it is possible to find an option when the magnetic moment of neutrino will be many orders of magnitude larger than the value (3.6) that we get from the easiest generalization of the Standard Model.

Terrestrial experimental limit on neutrino magnetic moment

GEMMA Collaboration (JINR+ITEP) in Dubna using the flux of neutrino ν_e from reactor obtained the following limit on the magnetic moment (2012):

$$\mu_{GEMMA} \leq 2{,}9 \cdot 10^{-11} \mu_B \tag{3.8}$$

They considered the scattering of neutrino on electron of the target. The cross-section is composed of two terms:

$$\frac{d\sigma}{dT} = \left.\frac{d\sigma}{dT}\right|_{SM} + \left.\frac{d\sigma}{dT}\right|_{\mu} \tag{3.9}$$

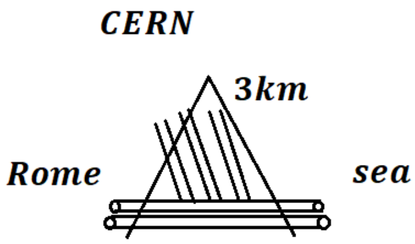
where the Standard Model weak contribution

$$\left.\frac{d\sigma}{dT}\right|_{SM} \sim G_F^2, \tag{3.10}$$

the Standard Model electromagnetic contribution

$$\left.\frac{d\sigma}{dT}\right|_{\mu} \sim \mu_\nu^2 \tag{3.11}$$

Of course both terms exist but they turned out to be beneath the background. And if the value is smaller than a particular number you immediately get the limit (3.8).



Pic. 3.5. Gran Sasso.

The same ideology has been used with the investigation of the solar fluxes. Borexino Collaboration (Gran Sasso, Italy) also obtained the limit on the magnetic moment. Gran Sasso is a mountain about 3 km high and about 150 km from Rome. There are two automobile tunnels inside this mountain about 11 km long that was constructed to make it easy for Romans to go to the sea. Famous Italian physicist de Kiki proposed to make also three caves A, B, C in the direction of CERN about 50 m long and about 20 m high. One of the famous experiments that confirmed the existence of neutrino oscillation phenomenon was located in one of these holes. They caught muon neutrinos coming from CERN along this hole and detected tau neutrinos in initial flux. The Borexino experiment located in one of the holes and contained of a big tank with scintillator that caught solar neutrinos coming from above the earth through this mountain. The limit on the magnetic moment obtained in Borexino experiment is slightly less:

$$\mu_{\text{Borexino}} \leq 2{,}8 \cdot 10^{-11} \mu_B \tag{3.12}$$

The best present limit has been obtained recently by XENONnT experiment also with solar neutrinos:

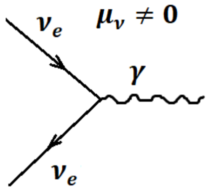
$$\mu_{\text{XENONnT}} \leq 6{,}2 \cdot 10^{-12} \mu_B \tag{3.13}$$

Astrophysical limit on neutrino magnetic moment

The first astrophysical limit on the magnetic moment of neutrino was obtained from studies of German scientist G. Raffelt in 1990:

$$\mu_{\text{ast.}} \leq 3 \cdot 10^{-12} \mu_B \tag{3.14}$$

It comes from the consideration of cooling of stars. So if the magnetic moment of neutrino is not zero $m_\nu \neq 0$ the following coupling appears and disturbs the observational picture of evolution of red giant stars (makes it faster):



Comparing this contribution to the observational data scientists get the limit (3.14). The result was updated in 2024 to the value: $1{,}5 \cdot 10^{-12} \mu_B$.

Millicharged neutrino

I would like to mention that the charge of neutrino can be also not zero $q_\nu \neq 0$. There are some generalizations of the Standard Model where it is possible to introduce without any contradiction with the theoretical framework that neutrino is a charged particle and called millicharged neutrino. The most severe bound on the charge of neutrino comes from the neutrality of the hydrogen atom

$$n \to p^+ + e^- + \nu_e \tag{3.15}$$

The charges of p^+ and e^- are measured with some accuracy. Comparing these charges we get the most severe bound on the charge of neutrino:

$$q_\nu < 10^{-21} e \tag{3.16}$$

In 2014 by Gemma experiment obtained the limit on the charge of neutrino. The cross-section of neutrino on electrons in Gemma data in case of millicharged neutrino contains an additional term:

$$\frac{d\sigma_{\nu-e}}{dT} = \left.\frac{d\sigma}{dT}\right|_{SM} + \left.\frac{d\sigma}{dT}\right|_{\mu_\nu \neq 0} + \left.\frac{d\sigma}{dT}\right|_{e-q_\nu}, \tag{3.17}$$

$$\left.\frac{d\sigma}{dT}\right|_{e-q_\nu} \sim q_\nu^2 \tag{3.18}$$

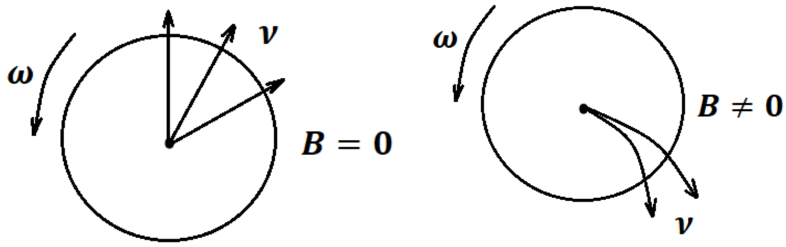
From the fact that (3.18) should be less than background we get

$$q_\nu < 1{,}3 \cdot 10^{-13} e \tag{3.19}$$

This result was considered the best and was published in the set of properties of neutrinos by the Particle Data Group collaboration. It is a group of about 100 most distinguished scientists that every two years publish the set of all properties of all interactions of all particles including neutrinos.

The best astrophysical bound also come from our paper "Nuclear Physics B" (2014) with my postgraduate student I.Tokarev:

$$q_\nu < 1{,}1 \cdot 10^{-19} e \tag{3.20}$$



Pic. 3.6. Neutrino star turning mechanism.

We considered the rotation of a pulsar in magnetic field. During the evolution most of the energy goes away with the flux of neutrinos. If there is no magnetic field neutrinos will escape perpendicular to the surface. If the magnetic field is not zero neutrinos will be move by the curved orbits. There is a momentum produced by millicharged charge neutrinos moving in a magnetic field and escaping the surface. It can disturb the rotation of the pulsar. We introduced such called neutrino star turning mechanism (NST). Due to NST there is a shift of the angle velocity of the pulsar $\Delta\omega$. We made a criteria that this shift should be less than observable rotational frequency of the pulsar:

$$\frac{\Delta\omega}{\omega_0} < 1 \tag{3.21}$$

This value depends on the charge of neutrino q_ν, therefore, we got the limit (3.20).

Neutrino could be considered as not charged particle but an effective superposition of clouds with opposite charges and in this case physicists introduce such property as a charge radius. The charge radius is the most easily accepted to be uh observed in experiment. It also contributes to the cross-section of neutrinos on electrons and even in the Standard Model the charge radius is not zero. Only one order of magnitude accuracy of the cross-section is needed to observe the charge radius of neutrino. I think that maybe in 10 years people will see the contribution of the charge radius to the cross-section.

Spin oscillations in transversal magnetic field

So the magnetic moment of neutrino is not zero. So in Lagrangian we have an additional term that describes an interaction of neutrinos with external electromagnetic field:

$$\mathcal{L} = \frac{1}{2}\mu\bar{\nu}\sigma_{ij}F^{ij}\nu + h.c. \tag{3.22}$$

We can decompose neutrino in two hiral or helicity states:

$$\begin{cases} \nu = \nu_L + \nu_R \\ \nu_L = \frac{1}{2}(1+\gamma_5)\nu \\ \nu_R = \frac{1}{2}(1-\gamma_5)\nu \end{cases}, \tag{3.23}$$

$\nu_L, \nu_R -$ left and right neutrinos. If we consider left and right neutrinos as different species we will see that there is a mixing in the interaction Lagrangian:

$$\mu\nu_R\sigma_{ij}F^{ij}\nu_L + \mu\nu_L\sigma_{ij}F^{ij}\nu_R \quad (3.24)$$

We can introduce a new phenomenon of mixing and oscillations of spin states of neutrinos in a transversal magnetic field $\vec{B}_\perp$. The transition between left and right neutrinos are completely determined by the interaction with transversal magnetic field in respect to neutrino propagation. The longitudinal part $\vec{B}_{||}$ only shifts the energy of each of neutrino species but doesn't produce a mixing. Also $\vec{B}_{||}$ is suppressed by an inverse Lawrence factor.

Transition magnetic moments

The term (3.22) in Lagrangian can be generalized when we consider different types of neutrino with different mass:

$$\mathcal{L} \sim \mu_{\alpha\beta}\nu_\alpha\sigma_{ij}F^{ij}\nu_\beta, \alpha \neq \beta \quad (3.25)$$

$\mu_{\alpha\beta} -$ off diagonal or transition magnetic moment.

There is another open fundamental question in neutrino physics whether neutrino Dirac or Majorana particle. For example, neutrino double β-decay probably can proceed only in case of Majorana neutrinos. Also there is a big difference for possible values of neutrino magnetic moment in case of Dirac and Majorana neutrinos. Obviously the diagonal magnetic moments for the case of Majorana neutrino are forbidden because of the CPT invariance theorem. So Majorana neutrinos can have only transition magnetic moments.

Lecture 4. Neutrino mass state evolution in constant magnetic field

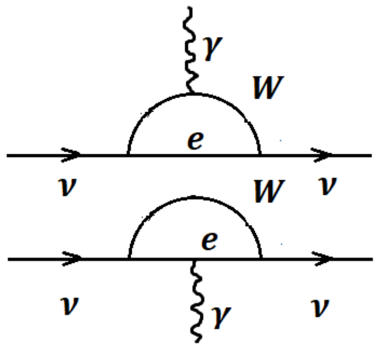
Neutrino properties

We know that the mass of neutrino is not zero $m_\nu \neq 0$. There are three mass states of neutrino ν_1, ν_2, ν_3 with non-zero masses m_1, m_2, m_3. According to the experimental results with solar and atmospheric neutrinos we know that at least two of three masses are non-zero and there are no two mass states with equal masses:

$$m_i \neq 0 \neq m_j, m_i \neq m_j \tag{4.1}$$

Each of flavor neutrinos $\nu^f : \nu_e, \nu_\mu, \nu_\tau$ is a superposition of the mass states. We can't introduce the masses for the flavor states of neutrino but we know how they interact with other particles according to the Standard Model. So when neutrinos propagate we should treat them as the mass states and when they interact with other particles we should use the flavor basis for their description.

If the mass of neutrino is not zero it immediately follow from the Standard Model Lagrangian that neutrino should have non-zero magnetic moments $\mu_{ii} \neq 0$. The non-zero magnetic moment of neutrino will interact with external magnetic field $\vec{B}$. Neutrino magnetic moment comes from the following diagrams:



Pic. 4.1. Neutrino interaction.

If the millicharge of neutrino is zero its magnetic moment is anomalous.

Neutrino spin oscillations in constant magnetic field

If magnetic moment of neutrino is not zero than there is an additional contribution to the Lagrangian of neutrino interaction. There is also a shift of energy of neutrinos due to this interaction and new mixing between left and right neutrinos. We will observe a phenomenon of neutrino spin oscillations in constant magnetic field. This effect is very important especially in astrophysics where magnetic fields are very strong.

For physical neutrinos $\nu^p = (\nu_1, \nu_2)$ we can write the Schrodinger evolution equation in the following form:

$$i\frac{d}{dt}\nu^p = H\nu^p \tag{4.2}$$

The Lagrangian of propagation of neutrino ν_1 or ν_2 equals

$$\mathcal{L} = -\mathcal{H} \tag{4.3}$$

The Hamiltonian in (4.2) equals

$$H = \begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix} \tag{4.4}$$

We can calculate the elements in (4.4) as

$$H = \begin{pmatrix} \langle\nu_1|\mathcal{H}|\nu_1\rangle & \langle\nu_1|\mathcal{H}|\nu_2\rangle \\ \langle\nu_2|\mathcal{H}|\nu_1\rangle & \langle\nu_2|\mathcal{H}|\nu_2\rangle \end{pmatrix} \tag{4.5}$$

We know that

$$\langle\nu_1, \nu_1\rangle = 1, \tag{4.6}$$

$$\langle\nu_2, \nu_1\rangle = 0, \tag{4.7}$$

therefore

$$H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}, \tag{4.8}$$

$$E_\alpha = \sqrt{m_\alpha^2 + |\vec{p}_\alpha|^2}, \alpha = 1,2 \tag{4.9}$$

Neutrino mixing and oscillations in matter

We know that photon γ in vacuum has zero mass $m_\gamma = 0$ and there is a simple equation between its energy and momentum:

$$E_\gamma = |\vec{p}_\gamma|, \tag{4.10}$$

$$v_\gamma = c = 1 \tag{4.11}$$

In presence of matter the mass of photon is not zero $m_\gamma \neq 0$ and

$$E_\gamma \neq |\vec{p}_\gamma|, \tag{4.12}$$

$$v_\gamma \neq 1 \tag{4.13}$$

The same happens with neutrinos in presence of matter, the energy and momentum relation also violates. We consider electrically neutral matter composed of electrons and protons having the same number density $n_e = n_p$ and neutrons with n_n. So we come to conclusion that the effective Hamiltonian of evolution of neutrino has the following matter term:

$$H_{matt.} = \begin{pmatrix} \sqrt{2}G_F\left(n_e - \frac{1}{2}n_n\right) & 0 \\ 0 & -\frac{1}{\sqrt{2}}G_F n_n \end{pmatrix} \tag{4.14}$$

Usually scientists consider only $\sqrt{2}G_F n_e$ value in $H_{matt.}$ Of course, in some extreme environments it may happen that $n_e \neq n_p$ and the number of muon could be non-zero $n_\mu \neq 0$. Also $H_{matt.}$ will be quite different if we consider neutrino mixing and oscillations between active and sterile neutrinos. Sterile neutrinos do not participate in any electro-weak interactions but they exist in nature and can show themself due to mixing.

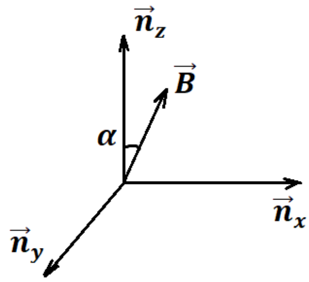
Dirac-Pauli equation

Let's write the Dirac equation that describes an interaction of massive neutrino having non-zero magnetic moment with external magnetic field:

$$\left(i\gamma_\mu \partial^\mu - m_i + \frac{\mu}{2}\sigma_{\mu\nu}F^{\mu\nu}\right)\nu_i(x) = 0, \tag{4.15}$$

$$\sigma_{\mu\nu} = \frac{i}{2}\left(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu\right) \tag{4.16}$$

We suppose that the millicharge of neutrino is zero $q_i = 0$ and we deal only with the anomalous magnetic moment of neutrino. We will consider constant magnetic field $F^{\mu\nu} \to \vec{B}$, $\vec{B} = (B_1, B_2, B_3)$.



Pic. 4.2. The vector magnetic field.

In this case we have the following expression for the tensor of electromagnetic fields:

$$F^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -B_3 & B_2 \\ 0 & B_3 & 0 & -B_1 \\ 0 & -B_2 & B_1 & 0 \end{pmatrix} \tag{4.17}$$

We introduce

$$\vec{\sigma}_{ij} = i\gamma_i\gamma_j \tag{4.18}$$

we can calculate the following term in (4.15):

$$\left.\frac{\mu}{2}\sigma_{\mu\nu}F^{\mu\nu}\right|_B = -\mu\vec{\Sigma}\vec{B}, \tag{4.19}$$

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \tag{4.20}$$

where $\mu \equiv \mu_{ii}$. We remember the expressions for the Pauli matrices:

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{4.21}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \tag{4.22}$$

$$\sigma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{4.23}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{4.24}$$

From the corresponding Dirac-Pauli Lagrangian we can derive the Hamiltonian that determines the evolution of neutrino particle:

$$\mathcal{L}_{DP} = -\mathcal{H}_{DP} \tag{4.25}$$

We should calculate the average of this Hamiltonian between different neutrino mass states $\langle \nu_i | \mathcal{H}_{DP} | \nu_j \rangle, i, j = 1,2$ and get the elements of the addition term to the effective evolution Hamiltonian:

$$\langle \nu_1 | \mathcal{H}_{DP} | \nu_1 \rangle = -\frac{\mu}{2} \int d^4x \nu_1^+ \gamma_0 \begin{pmatrix} \vec{\sigma}\vec{B} & 0 \\ 0 & \vec{\sigma}\vec{B} \end{pmatrix} \nu_2 \tag{4.26}$$

Where we use an exact neutrino state ν_i that corresponds to the mass, the momentum and to the energy as a solution of the vacuum Dirac equation:

$$\nu_1(m_1, p_1, E_1) = C_1 \sqrt{\frac{E_1 + m_1}{2E_1}} \begin{pmatrix} u_1 \\ \frac{\vec{\sigma}\vec{p}_1}{E_1 + m_1} u_1 \end{pmatrix} exp\{i(-E_1 t + \vec{p}_1 \vec{x})\}, \tag{4.27}$$

$$\nu_2(m_2, p_2, E_2) = C_2 \sqrt{\frac{E_2 + m_2}{2E_2}} \begin{pmatrix} u_2 \\ \frac{\vec{\sigma}\vec{p}_2}{E_2 + m_2} u_2 \end{pmatrix} exp\{i(-E_2 t + \vec{p}_2 \vec{x})\} \tag{4.28}$$

Where $u_\alpha, \alpha = 1,2$ is an additional characteristic that describes the spin states:

$$u_{\alpha, s=\frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{4.29}$$

$$u_{\alpha, s=-\frac{1}{2}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4.30}$$

In case when neutrino changes its type we can continue our calculations:

$$\frac{\mu_{12}}{2} \int d^4x \left(u_1^+, \frac{\vec{\sigma}\vec{p}_1}{E_1 + m_1} u_1^+ \right) \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \begin{pmatrix} u_2 \\ \frac{\vec{\sigma}\vec{p}_2}{E_2 + m_2} u_2 \end{pmatrix} \vec{B} \frac{\sqrt{(E_1 + m_1)(E_2 + m_2)}}{2\sqrt{E_1 E_2}} e^{i(-\Delta E t + \Delta p x)} \tag{4.31}$$

Without the loss of generality we suppose that both neutrino states move along axis z or the longitudinal component of magnetic field:

$$\vec{p}_1, \vec{p}_2 \parallel \vec{n}_z \parallel \vec{B}_{\parallel} \tag{4.32}$$

We should calculate the following term:

$$u_1^+(\vec{B}\vec{\sigma})u_2 - \frac{u_1^+(\vec{\sigma}\vec{p}_1)(\vec{\sigma}\vec{B})(\vec{\sigma}\vec{p}_2)u_2}{(E_1+m_1)(E_2+m_2)}$$
$$= u_1^+\left\{(\vec{\sigma}\vec{B}_{\parallel})\left(1-\frac{\vec{p}_1\vec{p}_2}{(E_1+m_1)(E_2+m_2)}\right)\right.$$
$$\left.+(\vec{\sigma}\vec{B}_{\perp})\left(1+\frac{\vec{p}_1\vec{p}_2}{(E_1+m_1)(E_2+m_2)}\right)\right\}u_2, \quad (4.33)$$

where we used the expression:

$$(\vec{\sigma}\vec{a})(\vec{\sigma}\vec{b}) = (\vec{a}\vec{b}) + i(\vec{\sigma}[\vec{a}\vec{b}]) \Rightarrow \quad (4.34)$$
$$(\vec{\sigma}\vec{p}_1)(\vec{\sigma}\vec{B})(\vec{\sigma}\vec{p}_2) = (\vec{p}\vec{B})(\vec{\sigma}\vec{p}) + i([\vec{p}\vec{B}]\vec{p}_1) - i(\vec{\sigma}[\vec{p}_1\vec{B}]\vec{p}_2)$$
$$= (\vec{p}\vec{B})(\vec{\sigma}\vec{p}) - i(\vec{\sigma}[\vec{p}_1\vec{B}]\vec{p}_2) \quad (4.35)$$

Let's agree that

$$|\vec{B}_{\parallel}| = B\cos\alpha, \quad (4.36)$$
$$|\vec{B}_{\perp}| = B\sin\alpha \quad (4.37)$$

So we should consider four possible combinations:

1) s_+s_+:

$$u^+_{1,s=\frac{1}{2}}(\vec{\sigma}\vec{B})u_{2,s=\frac{1}{2}} = (1,0)\vec{\sigma}\begin{pmatrix}1\\0\end{pmatrix}(\vec{B}_{\parallel}+\vec{B}_{\perp})$$
$$= (1,0)\sigma_3\begin{pmatrix}1\\0\end{pmatrix}B\cos\alpha + (1,0)\sigma_1\begin{pmatrix}1\\0\end{pmatrix}B\sin\alpha = B\cos\alpha \quad (4.38)$$

2) s_+s_-:

$$u^+_{1,s=\frac{1}{2}}(\vec{\sigma}\vec{B})u_{2,s=-\frac{1}{2}} = B\sin\alpha \quad (4.39)$$

3) s_-s_+:

$$u^+_{1,s=-\frac{1}{2}}(\vec{\sigma}\vec{B})u_{2,s=\frac{1}{2}} = -B\sin\alpha \quad (4.40)$$

4) s_-s_-:

$$u^+_{1,s=-\frac{1}{2}}(\vec{\sigma}\vec{B})u_{2,s=-\frac{1}{2}} = B\cos\alpha \quad (4.41)$$

Finally, for the average of the Hamiltonian between different neutrino mass states we get:

$$\langle\nu_1|\mathcal{H}_{DP}|\nu_2\rangle =$$
$$= -\frac{i}{2}\mu_{12}\int d^4x\begin{pmatrix}B\cos\alpha\cdot A & B\cos\alpha\cdot D\\ B\sin\alpha\cdot D & B-\cos\alpha\cdot A\end{pmatrix}\frac{\sqrt{(E_1+m_1)(E_2+m_2)}}{2\sqrt{E_1E_2}}e^{i(-\Delta Et+\Delta px)}, \quad (4.42)$$

$$A = 1 - \frac{\vec{p}_1\vec{p}_2}{(E_1+m_1)(E_2+m_2)}, \quad (4.43)$$

$$D = 1 + \frac{\vec{p}_1\vec{p}_2}{(E_1+m_1)(E_2+m_2)} \quad (4.44)$$

Let's calculate

$$A\frac{\sqrt{(E_1+m_1)(E_2+m_2)}}{2\sqrt{E_1E_2}}\bigg|_{\frac{m_1}{E_1}\ll 1,\frac{m_2}{E_2}\ll 1}=$$

$$=\frac{1}{2}\left(1+\frac{m_1}{2E_1}+\frac{m_2}{2E_2}-\left(1-\frac{m_1}{2E_1}\right)\left(1-\frac{m_2}{2E_2}\right)\right)=\frac{1}{2}\left(\frac{m_1}{E_1}+\frac{m_2}{E_2}\right) \quad (4.45)$$

where we used the expression:

$$\vec{p}_1\vec{p}_2=\sqrt{(E_1^2-m_1^2)(E_2^2-m_2^2)} \quad (4.46)$$

We introduce the transition gamma-factor as

$$\gamma_{12}^{-1}=\frac{1}{2}\left(\frac{m_1}{E_1}+\frac{m_2}{E_2}\right), \quad (4.47)$$

$$\gamma_{12}^{-1}|_{m_1=m_2}=\frac{1}{\gamma} \quad (4.48)$$

Let's calculate

$$D\frac{\sqrt{(E_1+m_1)(E_2+m_2)}}{2\sqrt{E_1E_2}}\bigg|_{\frac{m_1}{E_1}\ll 1,\frac{m_2}{E_2}\ll 1}\cong 1 \quad (4.49)$$

Finally, for the evolution of 4-component neutrino we get:

$$i\frac{d}{dt}\begin{pmatrix}\nu_{1,s=\frac{1}{2}}\\ \nu_{1,s=-\frac{1}{2}}\\ \nu_{2,s=\frac{1}{2}}\\ \nu_{2,s=-\frac{1}{2}}\end{pmatrix}=\begin{pmatrix}\mu_{11}\frac{B_{\parallel}}{\gamma_1} & \mu_{11}B_{\perp} & \mu_{12}\frac{B_{\parallel}}{\gamma_{12}} & \mu_{12}B_{\perp}\\ \mu_{11}B_{\perp} & -\mu_{11}\frac{B_{\parallel}}{\gamma_1} & \mu_{12}B_{\perp} & -\mu_{12}\frac{B_{\parallel}}{\gamma_{12}}\\ -\mu_{12}\frac{B_{\parallel}}{\gamma_{12}} & -\mu_{12}B_{\perp} & \mu_{22}\frac{B_{\parallel}}{\gamma_2} & \mu_{22}B_{\perp}\\ -\mu_{12}B_{\perp} & -\mu_{12}\frac{B_{\parallel}}{\gamma_{12}} & \mu_{22}B_{\perp} & -\mu_{22}\frac{B_{\parallel}}{\gamma_2}\end{pmatrix}\begin{pmatrix}\nu_{1,s=\frac{1}{2}}\\ \nu_{1,s=-\frac{1}{2}}\\ \nu_{2,s=\frac{1}{2}}\\ \nu_{2,s=-\frac{1}{2}}\end{pmatrix}, \quad (4.50)$$

where $\gamma_\alpha=\frac{E_\alpha}{m_\alpha}, \alpha=1,2$. The next step will be the transition of this result to the flavor states.

Lecture 5. Neutrino flavor state evolution in constant magnetic field

Helicity and chiral states of neutrino

We are going to continue the description of neutrino mixing and oscillations in constant magnetic field. We consider that magnetic field is composed of two components:

$$\vec{B} = \vec{B}_{\parallel} + \vec{B}_{\perp}, \tag{5.1}$$

$\vec{B}_{\parallel}$ – longitudinal magnetic field that is parallel to the neutrino momentum, $\vec{B}_{\perp}$ – transversal magnetic field that is perpendicular to the neutrino momentum. For simplicity we agreed to consider two mass states and two flavor states also accounting for two possible helicity states. We consider super relativistic neutrinos for which helicity and chirality are more or less equal to each other. As we know chirality is determined by operators:

$$P_{L,R} = \frac{1}{2}(1 \pm \gamma_5) \tag{5.2}$$

And helicity is determined by operators:

$$P_{\pm} = \frac{1}{2}\left(1 \mp \frac{\vec{\sigma}\vec{p}}{|\vec{p}|}\right) \tag{5.3}$$

In relativistic case when $m_\nu \approx 0$ the helicity and chiral states coincide with each other:

$$\nu_L \approx \nu_-, \tag{5.4}$$

$$\nu_R \approx \nu_+ \tag{5.5}$$

We suppose that we have two mass states ν_1, ν_2 with the corresponding masses m_1, m_2. Each of these states are characterized by spin orientation $s = \pm 1$. So we have four different states with two possible masses and spin orientations:

$$\nu_m = \begin{pmatrix} \nu_1^+ \\ \nu_1^- \\ \nu_2^+ \\ \nu_2^- \end{pmatrix} \tag{5.6}$$

Neutrino magnetic moment

We consider that magnetic moment of neutrino is not zero. The magnetic moment of neutrino with mass m_1 is μ_{11}, for neutrino with mass m_2 it is μ_{22}. We also should consider the hypothetical case when there is an interaction that changes the type of neutrino and introduce non diagonal or transition magnetic moments $\mu_{21} = \mu_{12}$.

The evolution equation for neutrino in constant magnetic field

For neutrinos (5.6) we can write the Schrodinger evolution equation:

$$i\frac{d}{dt}\nu_m = H_{eff}\nu_m \tag{5.7}$$

Effective evolution Hamiltonian is composed of two terms:

$$H_{eff} = H_{vac} + H_B, \tag{5.8}$$

$$H_{vac} = \begin{pmatrix} E_1 & 0 & 0 & 0 \\ 0 & E_1 & 0 & 0 \\ 0 & 0 & E_2 & 0 \\ 0 & 0 & 0 & E_2 \end{pmatrix}, \tag{5.9}$$

$$E_\alpha = \sqrt{m_\alpha^2 + |\vec{p}_\alpha|^2}, \alpha = 1,2 \tag{5.10}$$

We should notice that energy in vacuum doesn't depend on spin orientation. Component H_B corresponds to interaction of neutrino magnetic moment with magnetic field. We've already obtained the expression for component H_B. Finally, we get the following evolution equation:

$$i\frac{d}{dt}\nu_m = \begin{pmatrix} E_1 + \mu_{11}\frac{B_\parallel}{\gamma_{11}} & \mu_{11}B_\perp & \mu_{12}\frac{B_\parallel}{\gamma_{12}} & \mu_{12}B_\perp \\ \mu_{11}B_\perp & E_1 - \mu_{11}\frac{B_\parallel}{\gamma_{11}} & \mu_{12}B_\perp & -\mu_{12}\frac{B_\parallel}{\gamma_{12}} \\ \mu_{12}\frac{B_\parallel}{\gamma_{12}} & \mu_{12}B_\perp & E_2 + \mu_{22}\frac{B_\parallel}{\gamma_{22}} & \mu_{22}B_\perp \\ \mu_{12}B_\perp & -\mu_{12}\frac{B_\parallel}{\gamma_{12}} & \mu_{22}B_\perp & E_2 - \mu_{22}\frac{B_\parallel}{\gamma_{22}} \end{pmatrix}\begin{pmatrix} \nu_1^+ \\ \nu_1^- \\ \nu_2^+ \\ \nu_2^- \end{pmatrix}, \tag{5.11}$$

where

$$\gamma_{11} = \frac{E_1}{m_1}, \gamma_{22} = \frac{E_2}{m_2}, \tag{5.12}$$

$$\gamma_{12}^{-1} = \gamma_{21}^{-1} = \frac{1}{2}(\gamma_{11}^{-1} + \gamma_{22}^{-1}) \tag{5.13}$$

Often scientists don't take into account an interaction of neutrino with longitudinal magnetic field and neglect the term $\mu_{11}\frac{B_\parallel}{\gamma_{11}}$ because for relativistic neutrinos $\gamma_{11}^{-1} \ll 1$. Transversal magnetic field produces mixing between left and right neutrinos and longitudinal magnetic field slightly shifts the energy.

I would like to recall that we consider super relativistic neutrinos. It is an unsolved problem to consider mixing and oscillations for non-relativistic neutrinos. The most populated fraction of neutrinos is non-relativistic neutrinos that come from the earliest stages of Universe.

The term $\mu_{11}\frac{B_\parallel}{\gamma_{11}}$ enters the condition for resonance effect.

The longitudinal magnetic field also produces mixing between neutrino with different masses due to the term $\mu_{12}\frac{B_\parallel}{\gamma_{12}}$. For relativistic neutrinos $\gamma_{12}^{-1} \ll 1$ but never the less this new phenomenon exists.

The transition to flavor basis

We should convert all our results to flavor basis because all detectors are sensitive to flavor neutrinos. Flavor neutrino is a superposition of mass states:

$$\nu_f = U\nu_m \tag{5.14}$$

The mixing matrix:

$$U = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ -\sin\theta & 0 & \cos\theta & 0 \\ \mu_{12}B_\perp & -\sin\theta & 0 & \cos\theta \end{pmatrix} \tag{5.15}$$

Similarly with (5.6) for flavor neutrino we can write

$$\nu_f = \begin{pmatrix} \nu_e^+ \\ \nu_e^- \\ \nu_\mu^+ \\ \nu_\mu^- \end{pmatrix}, \tag{5.16}$$

$$\nu_e^\pm = \nu_1^\pm \cos\theta + \nu_2^\pm \sin\theta, \tag{5.17}$$

$$\nu_\mu^\pm = \nu_1^\pm \sin\theta + \nu_2^\pm \cos\theta, \tag{5.18}$$

where $\theta -$ vacuum mixing angle. The evolution equation for flavor neutrino:

$$i\frac{d}{dt}\nu_f = UHU^+\nu_f \tag{5.19}$$

We have to calculate new evolution Hamiltonian for flavor states

$$UHU^+ = UH_{vac}U^+ + UH_BU^+ \tag{5.20}$$

We've already calculated evolution Hamiltonian for flavor states in vacuum:

$$H'_{vac} = UH_{vac}U^+ = \left(|\vec{p}| + \frac{m_1^2 + m_2^2}{4|\vec{p}|}\right)I + \frac{\Delta m^2}{4|\vec{p}|}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}, \tag{5.21}$$

$$\Delta m^2 = m_2^2 - m_1^2 \tag{5.22}$$

From the point of view of probability of neutrino oscillations the term $\left(|\vec{p}| + \frac{m_1^2+m_2^2}{4|\vec{p}|}\right)I$ is not important.

Spin transitions can also appear due to weak interaction in case when neutrino propagates in moving matter and there is a transversal component of a matter current.

Now we should calculate the second term in (5.20):

$$H_B^f = UH_BU^+ = \begin{pmatrix} -\left(\frac{\mu}{\gamma}\right)_{ee} B_{\parallel} & \mu_{ee}B_\perp & -\left(\frac{\mu}{\gamma}\right)_{e\mu} B_{\parallel} & \mu_{e\mu}B_\perp \\ \mu_{ee}B_\perp & \left(\frac{\mu}{\gamma}\right)_{ee} B_{\parallel} & \mu_{e\mu}B_\perp & \left(\frac{\mu}{\gamma}\right)_{e\mu} B_{\parallel} \\ -\left(\frac{\mu}{\gamma}\right)_{e\mu} B_{\parallel} & \mu_{e\mu}B_\perp & -\left(\frac{\mu}{\gamma}\right)_{\mu\mu} B_{\parallel} & \mu_{\mu\mu}B_\perp \\ \mu_{e\mu}B_\perp & \left(\frac{\mu}{\gamma}\right)_{e\mu} B_{\parallel} & \mu_{\mu\mu}B_\perp & \left(\frac{\mu}{\gamma}\right)_{\mu\mu} B_{\parallel} \end{pmatrix}, \tag{5.23}$$

where

$$\left(\frac{\mu}{\gamma}\right)_{ee} = \frac{\mu_{11}}{\gamma_{11}}\cos^2\theta + \frac{\mu_{22}}{\gamma_{22}}\sin^2\theta + \frac{\mu_{12}}{\gamma_{12}}\sin 2\theta, \tag{5.24}$$

$$\left(\frac{\mu}{\gamma}\right)_{\mu\mu} = \frac{\mu_{11}}{\gamma_{11}}\sin^2\theta + \frac{\mu_{22}}{\gamma_{22}}\cos^2\theta - \frac{\mu_{12}}{\gamma_{12}}\sin 2\theta, \tag{5.25}$$

$$\left(\frac{\mu}{\gamma}\right)_{e\mu} = \frac{\mu_{12}}{\gamma_{12}}\cos 2\theta + \frac{1}{2}\left(\frac{\mu_{22}}{\gamma_{22}} - \frac{\mu_{11}}{\gamma_{11}}\right)\sin 2\theta, \tag{5.26}$$

μ_{ee} – an effective magnetic moment for electro neutrino:

$$\mu_{ee} = \mu_{11}\cos^2\theta + \mu_{22}\sin^2\theta + \mu_{12}\sin 2\theta, \tag{5.27}$$

μ_{ee} – an effective magnetic moment for muon neutrino:

$$\mu_{\mu\mu} = \mu_{11}\sin^2\theta + \mu_{22}\cos^2\theta - \mu_{12}\sin 2\theta, \tag{5.28}$$

$\mu_{e\mu}$ – a transition magnetic moment:

$$\mu_{e\mu} = \mu_{12}\cos 2\theta + \frac{1}{2}(\mu_{22} - \mu_{11})\sin 2\theta \tag{5.29}$$

We remember that the Lagrangian is proportional to

$$\mathcal{L} \sim \mu_{ij}\bar{\nu}_i\sigma_{\mu\nu}F^{\mu\nu}\nu_j \sim \mu_{i\neq j}\left(\bar{\nu}_L\sigma_{\mu\nu}F^{\mu\nu}\nu_R + \bar{\nu}_R\sigma_{\mu\nu}F^{\mu\nu}\nu_L\right) \tag{5.30}$$

If we consider left and right species belonging to different mass states than we obtain μ_{ij} or $\mu_{e\mu}$ for flavor states.

So we can consider different types of mixing and oscillations:

- $\nu_e^L \leftrightarrow \nu_e^R$ – spin oscillations
- $\nu_e^L \leftrightarrow \nu_\mu^R$ – spin-flavor oscillations
- $\nu_e^L \leftrightarrow \nu_\mu^L$ – flavor oscillations

The presence of matter shifts the oscillations.

Usually scientists consider flavor oscillations and spin-flavor oscillations separately. The probability of flavor oscillations depends on mass square difference and energy. The probability of spin-flavor oscillations depends on magnetic moment and strength of magnetic field. But recently I and my student A. Popov have showed that it is impossible to consider flavor and spin oscillations separately in the presence of magnetic field.

Lecture 6. Neutrino evolution in constant twisting magnetic field

Introduction

Last time we considered neutrino mixing and oscillations in magnetic field which has longitudinal and transversal components in respect to neutrino propagation and we derived an effective Hamiltonian of evolution accounting for matter and also for two possible spin orientations of the mass states.

1) Let's consider neutrino evolution between two neutrino states of equal masses and different helicities:

$$\nu_{1,s=1} \overset{B}{\leftrightarrow} \nu_{1,s=-1} \tag{6.1}$$

The corresponding evolution equation for this particular transition has the following form

$$i\frac{d}{dt}\begin{pmatrix}\nu_{1,s=1}\\ \nu_{1,s=-1}\end{pmatrix} = \frac{\mu_{11}}{2}\begin{pmatrix}\frac{B_{||}}{\gamma_1} & B_\perp \\ B_\perp & -\frac{B_{||}}{\gamma_1}\end{pmatrix}\begin{pmatrix}\nu_{1,s=1}\\ \nu_{1,s=-1}\end{pmatrix}, \tag{6.2}$$

where μ_{11} − the magnetic moment of neutrino ν_1 and

$$\gamma_1 = \frac{p_0}{m_1} \tag{6.3}$$

The interaction of magnetic moment with transversal and longitudinal magnetic field shifts the energy of neutrinos.

2) Now we should consider neutrino evolution between two neutrino states with different helicities and also different masses:

$$\nu_{1,s=1} \overset{B}{\leftrightarrow} \nu_{2,s=-1} \tag{6.4}$$

In this case the evolution equation looks like

$$i\frac{d}{dt}\begin{pmatrix}\nu_{1,s=1}\\ \nu_{2,s=-1}\end{pmatrix} = \frac{1}{2}\begin{pmatrix}\frac{\Delta m^2}{4p_0} + \mu_{11}\frac{B_{||}}{\gamma_1} & \mu_{12}B_\perp \\ \mu_{12}B_\perp & -\frac{\Delta m^2}{4p_0} - \mu_{22}\frac{B_{||}}{\gamma_2}\end{pmatrix}\begin{pmatrix}\nu_{1,s=1}\\ \nu_{2,s=-1}\end{pmatrix}, \tag{6.5}$$

where μ_{12} − the transition magnetic moment, Δm^2 − the mass square difference and

$$\gamma_2 = \frac{p_0}{m_2} \tag{6.6}$$

We consider neutrinos having more or less equal energies and equal momentums because the mass contribution is very small.

3) Let’s consider neutrino evolution between two neutrino states with different masses and equal spin orientation:

$$\nu_{1,s=1} \overset{B}{\leftrightarrow} \nu_{2,s=1} \tag{6.7}$$

In this case the evolution equation looks like

$$i\frac{d}{dt}\begin{pmatrix}\nu_{1,s=1}\\ \nu_{2,s=1}\end{pmatrix} = \frac{1}{2}\begin{pmatrix}\frac{\Delta m^2}{4p_0} + \mu_{11}\frac{B_{||}}{\gamma_1} & \mu_{12}\frac{B_{||}}{\gamma_{12}} \\ \mu_{12}\frac{B_{||}}{\gamma_{12}} & -\frac{\Delta m^2}{4p_0} + \mu_{22}\frac{B_{||}}{\gamma_2}\end{pmatrix}\begin{pmatrix}\nu_{1,s=1}\\ \nu_{2,s=1}\end{pmatrix}, \tag{6.8}$$

$$\gamma_{12}^{-1} = \frac{1}{2}(\gamma_1^{-1} + \gamma_2^{-1}) \tag{6.9}$$

We see that mixing between two different mass states with the same spin orientation appears in case when the transition magnetic moment is not zero $\mu_{12} \neq 0$ and also due to longitudinal magnetic field. The same effect will appear for flavor neutrinos. There will be an additional mixing in the flavor basis, in addition to mixing that is governed by the mixing angle.

Twisting magnetic field

Let's consider mixing and oscillations in constant twisting magnetic field. This field is perpendicular in respect to neutrino propagation, constant in time and rotates in space:

$$\vec{B} = \vec{B}_\perp, \tag{6.10}$$

$$B = B_\perp e^{i\phi(t)} \tag{6.11}$$

Such form turned out to be applicable in some models of the structure of the electromagnetic field, in particular, the structure of magnetic field on the Sun. I would like to mention that this approach has been used to describe another phenomenon that was proposed and investigated in my papers about 20 years ago. We assumed that probably it is possible to solve the same problem not in constant magnetic field but in electromagnetic field of an electromagnetic wave. Then we went further and investigated some other very interesting phenomena related to neutrino spin evolution in external environments simultaneously including the presence of electromagnetic fields and matter that can be at rest or moving with a relativistic speed.

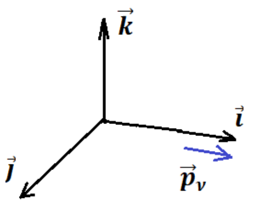


Fig. 6.1. The direction of neutrino propagation.

In system of axes shown in the fig.6.1 we can write

$$\vec{B} = \vec{j}B_2 + \vec{k}B_3 \tag{6.12}$$

We consider the following neutrino state

$$\nu = \begin{pmatrix}\nu_e^L\\ \nu_\mu^R\end{pmatrix} \tag{6.13}$$

We will derive the evolution equation and solve it for this particular neutrino state. We should account for all possible magnetic moments $\mu_{ij} \to \mu_{11} \equiv \mu_1, \mu_{22} \equiv \mu_2, \mu_{12} = \mu_{21}$ introduced in the mass states. If you remember we also derived the effective magnetic moments for flavor neutrinos that contain the initial fundamental magnetic moments introduced for the mass states. We introduce mixing as

$$\begin{cases} \nu_e = \nu_1 \cos\theta + \nu_2 \sin\theta \\ \nu_\mu = -\nu_1 \sin\theta + \nu_2 \cos\theta \end{cases} \tag{6.14}$$

The problem of evolution between different flavor state in twisting magnetic field and matter

We are going to solve the following evolution equation:

$$i\frac{d}{dt}\begin{pmatrix} \nu_e^L \\ \nu_\mu^R \end{pmatrix} = \left(\widetilde{H} + H_B^f\right)\begin{pmatrix} \nu_e^L \\ \nu_\mu^R \end{pmatrix}, \tag{6.15}$$

$$\widetilde{H} = H'_{vac} + H_{matt}, \tag{6.16}$$

where $H_B^f -$ the Hamiltonian of interaction of magnetic moment with external magnetic field in flavor basis, $H'_{vac} -$ the evolution Hamiltonian in vacuum after transition to the flavor basis, $H_{matt} -$ the evolution Hamiltonian in matter that has been introduced in flavor basis. We use the normal matter composed of electrons, protons and neutrons with equal number of electrons and protons:

$$n_e = n_p, n_n \tag{6.17}$$

We also suppose that there are no electric currents.

Initially in mass basis or physical basis we have the following equation:

$$i\frac{d}{dt}\nu^p = H\nu^p, \tag{6.18}$$

$$H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}, \tag{6.19}$$

$$E_\alpha = \sqrt{m_\alpha^2 + |\vec{p}_\alpha|^2} \approx |\vec{p}|^2 + \frac{m_\alpha^2}{2|\vec{p}|} + \cdots, \alpha = 1,2 \tag{6.20}$$

The Hamiltonian can be written in the following form

$$H = \left(|\vec{p}|^2 + \frac{m_1^2 + m_2^2}{4|\vec{p}|}\right)\hat{I} - \frac{\Delta m^2}{4|\vec{p}|}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{6.21}$$

$$\Delta m^2 = m_2^2 - m_1^2 \tag{6.22}$$

In case of mixing between two different mass states and flavor states we have

$$\nu^f = U\nu^p, \tag{6.23}$$

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{6.24}$$

For the evolution of flavor states we can write the following equation:

$$i\frac{d}{dx}\nu^f = UHU^+\nu^p, \tag{6.25}$$

$$H'_{vac} = UHU^+ \tag{6.26}$$

We put x instead of t in (6.25) because for the relativistic neutrinos the evolution in space equals to evolution in time.

After we introduce matter there are two contributions: neutral current NC and charged current CC. Neutral current and charged current contributions can be described by the following diagrams (fig. 6.2, fig. 6.3).

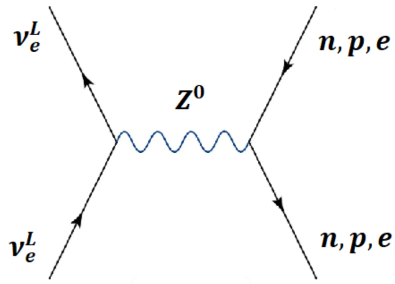


Fig. 6.2. Neutral current.

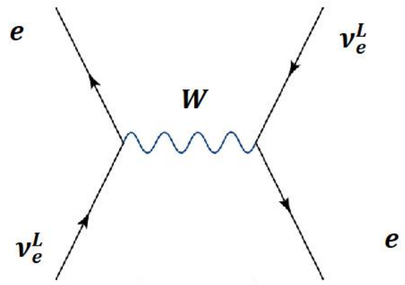


Fig. 6.3. Charged current.

We have left-handed electron neutrino ν_e^L that does interact with environment through neutral and charged currents. We also have right-handed muon neutrino ν_μ^R that due to the structure of electro-weak Lagrangian is sterile in case of massless neutrinos. If the mass is very small in respect to energy it's nearly sterile. So we suppose that muon neutrino is a sterile particle, it means that none of these contributions NC and CC appears for right-handed muon neutrino. This is quite a different situation in respect to what we discussed before when we considered the mixing and oscillations between flavor neutrinos without change of spin orientation. At that time we considered right-handed muon neutrino as an active particle. Summing it all up we can write the effective matter potential for left-handed electron neutrino due to neutral current interaction:

$$V_{\nu_e^L}^{NC} = -\frac{1}{\sqrt{2}}G_F n_n \tag{6.27}$$

Protons and electrons have the opposite charges and their contributions kill each other. The effective matter potential for left-handed electron neutrino due to charged current interaction:

$$V_{\nu_e^L}^{CC} = \sqrt{2} G_F n_e \tag{6.28}$$

The potential for right-handed muon neutrino is zero:

$$V_{\nu_\mu^R} = 0 \tag{6.29}$$

Let's write the difference of potentials that is important for oscillation probability calculations:

$$\Delta V_{\nu_e^L \nu_\mu^R}\Big|_{n_e = n_p} = \sqrt{2} G_F \left(n_e - \frac{n_n}{2}\right) \tag{6.30}$$

If we sum up all that we have discussed above we can write the effective evolution equation for the considered neutrino state:

$$i\frac{d}{dt}\begin{pmatrix} \nu_e^L \\ \nu_\mu^R \end{pmatrix} = \begin{pmatrix} E_L & \mu_{e\mu} B e^{-i\phi(t)} \\ \mu_{e\mu} B e^{i\phi(t)} & E_R \end{pmatrix} \begin{pmatrix} \nu_e^L \\ \nu_\mu^R \end{pmatrix}$$

$$= \left\{ \left(|\vec{p}|^2 + \frac{m_1^2 + m_2^2}{4|\vec{p}|} \right) \hat{I} + \begin{pmatrix} -\frac{\Delta m^2}{4p_0} \cos 2\theta + V_{\nu_e^L} & \mu_{e\mu} B e^{-i\phi(t)} \\ \mu_{e\mu} B e^{i\phi(t)} & \frac{\Delta m^2}{4p_0} \cos 2\theta \end{pmatrix} \right\} \begin{pmatrix} \nu_e^L \\ \nu_\mu^R \end{pmatrix} \tag{6.31}$$

$\mu_{e\mu} -$ transition magnetic moment in flavor basis. The only problem we have is the term $\phi = \phi(t)$. This problem can be formulated unitary transformation from flavor basis to another one for which the term $e^{-i\phi(t)}$ will disappear and an additional potential will appear. As soon as two bases are related by unit transformation the solution for the probability will be the same. So let's consider new basis

$$\nu \rightarrow \nu', \tag{6.32}$$

$$\nu = U\nu', \tag{6.33}$$

$$U = \begin{pmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{pmatrix} \tag{6.34}$$

We will consider evolution in terms of new introduced basis indicated by $\quad '$. New evolution equation looks like

$$i\left[\frac{i}{2}\begin{pmatrix} -e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{pmatrix} \dot{\phi}\nu' + \begin{pmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{pmatrix} \frac{d}{dt}\nu' \right] = \widetilde{H} \begin{pmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{pmatrix} \nu' \tag{6.35}$$

We can introduce the matrices:

$$U^+ = \begin{pmatrix} e^{i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{pmatrix}, \tag{6.36}$$

$$U' = \begin{pmatrix} -e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{pmatrix} \tag{6.37}$$

Let's multiply (6.35) by matrix U^+ from the left side:

$$i\frac{d}{dt}\nu' = \left(U^+\widetilde{H}U + \frac{\dot{\phi}}{2}U^+U'\right)\nu', \tag{6.38}$$

$$U^+U' = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} \tag{6.39}$$

The function $\phi(t)$ appears in form of derivative $\dot{\phi}$. We see an addition to effective Hamiltonian in (6.38), so the effective energy is changing. To go further we need to calculate the following term

$$U^+\widetilde{H}U = \begin{pmatrix} e^{i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{pmatrix}\begin{pmatrix} A & \mu B e^{-i\phi} \\ \mu B e^{i\phi} & C \end{pmatrix}U$$

$$= \begin{pmatrix} Ae^{i\phi/2} & \mu B e^{-i\phi/2} \\ \mu B e^{i\phi/2} & Ce^{-i\phi/2} \end{pmatrix}\begin{pmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{pmatrix} = \begin{pmatrix} A & \mu_{e\mu}B \\ \mu_{e\mu}B & C \end{pmatrix} = \widetilde{H}\big|_{\phi=0} \tag{6.40}$$

Finally, we get

$$i\frac{d}{dt}\begin{pmatrix} \nu_e^L \\ \nu_\mu^R \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4p_0}\cos 2\theta + \frac{V_{\nu_e^L}}{2} - \frac{\dot{\phi}}{2} & \mu_{e\mu}B \\ \mu_{e\mu}B & \frac{\Delta m^2}{4p_0}\cos 2\theta - \frac{V_{\nu_e^L}}{2} + \frac{\dot{\phi}}{2} \end{pmatrix}\begin{pmatrix} \nu_e^L \\ \nu_\mu^R \end{pmatrix} \tag{6.41}$$

We introduce the σ-matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{6.42}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Rightarrow \tag{6.43}$$

$$i\frac{d}{dt}\begin{pmatrix} \nu_e^L \\ \nu_\mu^R \end{pmatrix} = \left(-\frac{\Delta_{LR}}{4p_0}\sigma_3 + \mu_{e\mu}B\sigma_1\right)\begin{pmatrix} \nu_e^L \\ \nu_\mu^R \end{pmatrix}, \tag{6.44}$$

where

$$\Delta_{LR} = \Delta m^2 \cos 2\theta - 2p_0 V_{\nu_e^L} + 2p_0\dot{\phi} \tag{6.45}$$

So in addition to real potential we a mimic potential due to the effect of rotation in space of the transversal magnetic field.

Lecture 7. Spin-flavor mixing and oscillations of neutrino in constant twisting magnetic field

The evolution equation for neutrinos with non-zero magnetic moment in constant twisting magnetic field

Last time we have derived the evolution equation for neutrinos with non-zero magnetic moment $\mu \neq 0$ of two different flavors with different spin orientations in constant twisting magnetic field

$$B = |\vec{B}| e^{i\phi(t)} \tag{7.1}$$

We have introduced the magnetic moments of the mass states of neutrinos and electromagnetic interactions for the mass states. Then we have made the transition to the flavor state and get the following equation:

$$i\frac{d}{dt}\begin{pmatrix} \nu_e^L \\ \nu_\mu^R \end{pmatrix} = \left(-\frac{\Delta_{LR}}{4p_0}\sigma_3 + \mu_{e\mu}B\sigma_1\right)\begin{pmatrix} \nu_e^L \\ \nu_\mu^R \end{pmatrix}, \tag{7.2}$$

$$\Delta_{LR} = \Delta m^2 \cos 2\theta - 2p_0 V_{\nu_e^L} + 2p_0\dot{\phi}, \tag{7.3}$$

$$\Delta m^2 = m_2^2 - m_1^2, \tag{7.4}$$

where $\mu_{e\mu}$ − the transition magnetic moment in flavor basis, $V_{\nu_e^L}$ − the effective potential of left-handed electron neutrino. We forget to mention that we also consider interaction with matter composed of electrons, protons and neutrons. The solution of the equation (7.2) looks like

$$\begin{pmatrix} \nu_e^L(x) \\ \nu_\mu^R(x) \end{pmatrix} = e^{-i\left[-\frac{\Delta_{LR}}{4p_0}\sigma_3 + \mu_{e\mu}B\sigma_1\right]x}\nu(0), \tag{7.5}$$

where the initial state of neutrino

$$\nu(0) = \begin{pmatrix} \nu_e^L(0) \\ \nu_\mu^R(0) \end{pmatrix} \tag{7.6}$$

We will use the following well-known formula:

$$e^{-i(\vec{\sigma}\vec{a})\phi} = \cos\phi \; i\vec{\sigma}\vec{a}\sin\phi \tag{7.7}$$

Let's introduce

$$\Omega^2 = \left(\mu_{e\mu}B\right)^2 + \left(\frac{\Delta_{LR}}{4p_0}\right)^2, \tag{7.8}$$

$$\vec{k} = \left(\mu_{e\mu}B, 0, -\frac{\Delta_{LR}}{4p_0}\right) \tag{7.9}$$

It is easy to notice that

$$\vec{k}^2 = \Omega^2, \tag{7.10}$$

$$\vec{k}\vec{\sigma} = \mu_{e\mu} B\sigma_1 - \frac{\Delta_{LR}}{4p_0}\sigma_3 \tag{7.11}$$

The solution of the neutrino evolution equation will be

$$\nu(x) = \begin{pmatrix} \nu_e^L(x) \\ \nu_\mu^R(x) \end{pmatrix} = e^{-i\Omega x\left(\frac{\vec{k}}{\Omega}\vec{\sigma}\right)} \begin{pmatrix} \nu_e^L(0) \\ \nu_\mu^R(0) \end{pmatrix}$$

$$= \left[\cos\Omega x - \frac{i}{\Omega}\left(\frac{\Delta_{LR}}{4p_0}\sigma_3 - \mu_{e\mu} B\sigma_1\right)\sin\Omega x\right]\begin{pmatrix} \nu_e^L(0) \\ \nu_\mu^R(0) \end{pmatrix} \tag{7.12}$$

The probability of neutrino spin-flavor oscillations

The probability of oscillations between left-handed electron neutrino and right-handed muon neutrino:

$$P_{\nu_e^L \to \nu_\mu^R}(x) = \left|\langle \nu_\mu^R | \varphi_e^+(x)\rangle\right|^2 = \left|(0,1)\left(\cos\Omega x - \frac{i}{\Omega}\left(\frac{\Delta_{LR}}{4p_0}\sigma_3 - \mu_{e\mu} B\sigma_1\right)\sin\Omega x\right)\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right|^2$$

$$= \frac{\left(\mu_{e\mu}B\right)^2}{\Omega^2}\sin^2\Omega x = \frac{\left(\mu_{e\mu}B\right)^2}{\left(\mu_{e\mu}B\right)^2 + \left(\frac{\Delta_{LR}}{4p_0}\right)^2}\sin^2\Omega x, \tag{7.13}$$

where

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{7.14}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{7.15}$$

We obtained the probability for neutrino spin-flavor oscillations (7.13). This is a new phenomenon in respect to flavor oscillations. As it was shown very recently in our papers that it is impossible to consider neutrino spin and flavor oscillations in magnetic field separately. In presence of magnetic field the amplitudes of oscillations depend on the magnetic moment and the magnetic field. If there is no magnetic field the mixing between left and right neutrinos does not exist. But about 20 years ago in our paper I have proposed another new phenomenon that the spin mixing and oscillations can be generated without any need of non-zero magnetic moment and the presence of magnetic field. Such mixing and oscillations between left and right neutrinos can be produced by weak interaction of neutrinos with the moving matter in case when there is a non-zero transversal component of matter in respect to neutrino propagation. I had proposed this idea in my papers in 2004. And then about 10 years after that other people had used it to study astrophysical neutrino fluxes that were produced under the influence of extreme astrophysical conditions where dense fluxes of rotating matter existed.

Let's compare the probabilities of flavor oscillations and spin-flavor oscillations. In case of two flavor and mass states the probability of flavor oscillations:

$$P_{\nu_e \to \nu_\mu}(x) = \sin^2 \tilde{\theta} \sin^2 \left(\pi \frac{x}{L_m}\right), \tag{7.16}$$

where $\theta -$ the mixing angle in vacuum, $L_m -$ the length of oscillations in matter. In presence of matter we should replace θ with $\tilde{\theta} -$ the effective mixing angle in matter. We can write for the oscillation length

$$L_m = \frac{2\pi}{\tilde{E}_1 - \tilde{E}_2} = \frac{2\pi}{\sqrt{\left(\frac{\Delta m^2}{2p_0}\cos 2\theta\right)^2 - \left(\sqrt{2}G_F n_e\right)^2 + \left(\frac{\Delta m^2}{2p_0}\sin 2\theta\right)^2}}, \tag{7.17}$$

Where $\tilde{E}_1, \tilde{E}_2 -$ the effective energies accounting for the scattering of neutrinos on particles in matter. I would like to remind you that we consider the normal matter composed of electrons, protons and neutrons with equal number of electrons and protons:

$$n_e = n_p, n_n \tag{7.18}$$

The probability of spin-flavor oscillations:

$$P_{\nu_e^L \to \nu_\mu^R}(x) = \sin^2 \beta \sin^2 \Omega x, \tag{7.19}$$

$$\sin^2 \beta = \frac{\left(\mu_{e\mu} B\right)^2}{\left(\mu_{e\mu} B\right)^2 + \left(\frac{\Delta_{LR}}{4p_0}\right)^2} \tag{7.20}$$

The amplitude of flavor oscillations equals to $\sin^2 \tilde{\theta}$, the amplitude of spin-flavor oscillations equals to $\sin^2 \beta$. From (7.19) we can derive the probability of spin oscillations (when the flavor doesn't change) in magnetic field in matter:

$$P_{\nu_e^L \to \nu_e^R}(x) = ? \tag{7.21}$$

I would like to mention that in case when the amplitude $\sin^2 \beta$ gets its maximum we observe the resonance effect. It could be achieved when

$$\frac{\left(\mu_{e\mu} B\right)^2}{\left(\mu_{e\mu} B\right)^2 + \left(\frac{\Delta_{LR}}{4p_0}\right)^2} = 1 \Rightarrow \tag{7.22}$$

$$\Delta_{LR} \to 0 \tag{7.23}$$

or

$$\mu_{e\mu} B \gg \left|\frac{\Delta_{LR}}{4p_0}\right| \tag{7.24}$$

The resonance effect for spin-flavor oscillations was first theoretically discovered in 1988 in two different papers by E. Akhmedov and Lim and Marciano.

I would like to recall the Lagrangian that describes the neutrino magnetic moment interaction with external magnetic field:

$$L_{em} = \mu_{e\mu}\bar{\nu}_e\sigma_{\mu\nu}F^{\mu\nu}\nu_\mu = \mu_{e\mu}\bar{\nu}_e^L\sigma_{\mu\nu}F^{\mu\nu}\nu_\mu^R + \mu_{e\mu}\bar{\nu}_\mu^R\sigma_{\mu\nu}F^{\mu\nu}\nu_e^R \tag{7.25}$$

We suppose that electron and muon neutrinos are superpositions of corresponding left-handed and right-handed neutrinos:

$$\nu_e = \nu_e^L + \nu_e^R, \tag{7.26}$$

$$\nu_\mu = \nu_\mu^L + \nu_\mu^R \tag{7.27}$$

We can write the evolution equations for left-handed electron and muon neutrinos:

$$\begin{cases} i\dfrac{d\nu_e^L}{dx} = -\dfrac{\Delta_{LR}}{4p_0}\nu_e^L + \mu_{e\mu}B\nu_\mu^R \\ i\dfrac{d\nu_\mu^L}{dx} = \dfrac{\Delta_{LR}}{4p_0}\nu_\mu^L + \mu_{e\mu}B\nu_e^R \end{cases} \tag{7.28}$$

Neutrino mixing and oscillations in moving matter

Now we should consider the phenomenon of neutrino mixing and oscillations due to interaction with the moving matter. So there are neutrino mixing and oscillations $\nu_L \leftrightarrow \nu_R$ due to transversal matter current $j_\perp$. This is a weak interaction determined by constant G_F. In this case there is no need in non-zero magnetic moment or magnetic field. Spin and spin-flavor oscillations can be produced.

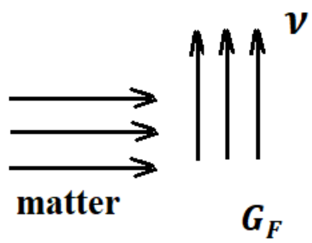


Fig. 7.1. Transversal matter current.

At first this phenomenon was predicted in my paper in 2004. The oscillations also can be produced if matter is at rest but polarized in direction transversal to neutrino propagation. Another two papers were published in 2001 and 2002 by me and Lobanov where we proposed a quasi-classical treatment of neutrino spin evolution in the presence of external magnetic field or matter. We generalized the classical equation of spin evolution in electrodynamics or Bargmann-Michele-Telegdi equation adding the weak interaction of neutrinos with matter.

$$\frac{d\vec{s}}{dt} = \frac{2}{\gamma}\left[\vec{s} \times \left(\vec{B}_0 + \vec{H}_0\right)\right], \tag{7.29}$$

$$\gamma = \frac{p_0}{m}, \tag{7.30}$$

where $\vec{s}$ – neutrino spin vector, $\vec{B}_0$ – magnetic field in the rest frame of neutrino that can be expressed in terms of electromagnetic field in laboratory frame:

$$\vec{B}_0 = \gamma\left(\vec{B}_\perp + \frac{1}{\gamma}\vec{B}_{||} + \sqrt{1 - \frac{1}{\gamma^2}}\left[\vec{E}_\perp \times \frac{\vec{\beta}}{\beta}\right]\right), \tag{7.31}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \tag{7.32}$$

where $\vec{\beta}$ – neutrino velocity in laboratory frame. We can see that the longitudinal part of magnetic field is neglected by $\gamma \gg 1$. The matter term in laboratory frame:

$$\vec{M} = \vec{M}_{0||} + \vec{M}_{0\perp}, \tag{7.33}$$

$$\vec{M}_{0||} = \gamma\vec{\beta}\frac{n_0}{\sqrt{1-v_e^2}}\left\{\rho_e^{(1)}\left(1 - \frac{v_e\vec{\beta}}{1-\frac{1}{\gamma^2}}\right) - \frac{\rho_e^{(2)}}{1-\frac{1}{\gamma^2}}\left[\vec{\xi}_e\vec{\beta}\sqrt{1-v_e^2} + \vec{\xi}_e\vec{v}_e - \frac{\left(\vec{\beta}\vec{v}_e\right)}{1+\sqrt{1-v_e^2}}\right]\right\}, \tag{7.34}$$

where n_0 – number density of electrons in laboratory frame, v_e – speed of electrons, $\vec{\xi}_e$ – polarization of matter. For simplicity let's don not account the polarization effect and neglect corresponding terms from (7.34). The mixing and oscillations between left-handed and right-handed neutrino states appear due to neutrino interaction with the transversal moving matter that can be described by $\vec{M}_{0\perp}$:

$$\vec{M}_{0\perp} = -\frac{n_0}{\sqrt{1-v_e^2}}\left\{\vec{v}_{e\perp}\rho_e^{(1)} + \theta\vec{\xi}_e\right\} \tag{7.35}$$

Only the first term in (7.35) is important to us. Where

$$\rho_e^{(1)} = \frac{G_F}{2\sqrt{2}\mu} \tag{7.36}$$

You shouldn't be afraid about the appearance of magnetic moment; it will be canceled out in the following calculations.

But in our papers we didn't discovered the effect of mixing and oscillations due to interaction with the transversal moving matter because we supposed that all was suppressed by γ-factor. The formulas (7.34) and (7.35) haven't been used in our papers.

The evolution equation in this case will look like

$$i\frac{d}{dt}\begin{pmatrix}\nu_e^L\\ \nu_\mu^R\end{pmatrix} = \mu\begin{pmatrix}\frac{1}{\gamma}\left|\vec{M}_{0_{||}}+\vec{B}_{0_{||}}\right| & \left|\vec{B}_\perp+\frac{1}{\gamma}\vec{M}_{0_\perp}\right| \\ \left|\vec{B}_\perp+\frac{1}{\gamma}\vec{M}_{0_\perp}\right| & -\frac{1}{\gamma}\left|\vec{M}_{0_{||}}+\vec{B}_{0_{||}}\right|\end{pmatrix}\begin{pmatrix}\nu_e^L\\ \nu_\mu^R\end{pmatrix}, \tag{7.37}$$

where $\vec{M}_{0_{||}}, \vec{M}_{0_\perp} \sim 1/\mu$. We can consider the case when $B=0$ or $\mu=0$ and only moving matter remains in (7.37). The probability of oscillations between left-handed and right-handed electron neutrino equals

$$P_{\nu_e^L\to\nu_\mu^R}(x) = \sin^2 2\theta_{eff}\sin^2\frac{\pi x}{L_{eff}} \tag{7.38}$$

We get the usual expression for the probability of neutrino oscillations, where

$$\sin^2 2\theta_{eff} = \frac{E_{eff}^2}{E_{eff}^2+\Delta_{eff}^2}, \tag{7.39}$$

$$E_{eff} = \mu\left(\vec{B}_\perp+\frac{1}{\gamma}\vec{M}_{0_\perp}\right), \tag{7.40}$$

$$\Delta_{eff} = \frac{\mu}{\gamma}\left(\vec{M}_{0_{||}}+\vec{B}_{0_{||}}\right) \tag{7.41}$$

We can see that when $B=0$ or $\mu=0$ there is non-zero probability of neutrino oscillations.

In about 2015 people started to use this phenomenon of neutrino oscillations in the transversal moving matter. The main point is that ν_e^L is an active neutrino but ν_μ^R is a sterile neutrino and it can freely propagate from very hot inner parts of the star.

We obtained the result only for spin oscillations but not for flavor ones. Next time we will consider quantum approach to this effect where we it is possible consider the change of flavor.

Lecture 8. Neutrino spin-flavor oscillations in moving matter

Neutrino mixing and oscillations due to transversal matter current

Let's continue our discussion of the effect of neutrino mixing and oscillations due to transversal matter currents. We considered before the phenomenon of spin and spin-flavor oscillations due to interaction of non-zero magnetic moment of neutrino with transversal magnetic field. But there are also spin and spin-flavor oscillations that appear due to weak interaction of neutrinos with the moving matter. This new phenomenon for the first time was proposed about 20 year ago in our paper. More recently about 5 years ago we made more complete derivation within the quantum approach. In the previous lecture we considered this affect using the quasi-classical approach.

For simplicity we consider only two flavor states with different helicities:

$$\nu_f = \left(\nu_e^+, \nu_e^-, \nu_\mu^+, \nu_\mu^-\right) \tag{8.1}$$

We suppose that neutrinos are relativistic, therefore

$$m_\nu \ll p_0^\nu \tag{8.2}$$

Neutrinos move in matter composed of neutrons. We also account for the effect of non-zero matter current

$$\vec{j}_\mu \neq 0 \tag{8.3}$$

The transversal component of this current $\vec{j}_\perp \perp \vec{p}_\nu$ produces the mixing and oscillations between different neutrino spin states. The Lagrangian of neutrino interaction looks like

$$L_{int} = -f^\mu \sum_{l=e,\mu} \bar{\nu}_e(x)\gamma_\mu \frac{1+\gamma_5}{2}\nu_e(x), \tag{8.4}$$

$$f^\mu = -\frac{G_F}{\sqrt{2}} n(1, \vec{v}), \tag{8.5}$$

where the number density of neutrons in the laboratory frame

$$n = \frac{n_0}{\sqrt{1-\vec{v}^2}}, \tag{8.6}$$

the velocity of matter composed of neutrons in the laboratory frame

$$\vec{v} = (v_1, v_2, v_3) \tag{8.7}$$

If we express each flavor state in terms of mass states we can rewrite (8.4) as the sum over mass states:

$$L_{int} = -f^\mu \sum_{i=1,2} \bar{\nu}_i(x)\gamma_\mu \frac{1+\gamma_5}{2}\nu_i(x) \tag{8.8}$$

Flavor states in terms of mass states look like

$$\begin{cases} \nu_e^{\pm} = \nu_1^{\pm}\cos\theta + \nu_2^{\pm}\sin\theta \\ \nu_\mu^{\pm} = -\nu_1^{\pm}\sin\theta + \nu_2^{\pm}\cos\theta \end{cases} \tag{8.9}$$

I would like to recall the expression for the neutrino evolution equation in the flavor basis:

$$i\frac{d}{dt}\nu_f = (H_0 + \Delta\overline{H}_{SM})\nu_f, \tag{8.10}$$

where H_0 describes the effect of neutrino propagation in non-moving matter, $\Delta\overline{H}_{SM}$ describes the neutrino propagation in moving matter. For the first time the value $\Delta\overline{H}_{SM}$ was calculated in our paper in 2016, more complete result was obtained in 2018 by Pustoshny and me.

The addition to the effective Hamiltonian

In more general case the addition to the effective Hamiltonian could be described in the following way:

$$\Delta\overline{H}_{SM} = \begin{pmatrix} \Delta_{ee}^{++} & \Delta_{ee}^{+-} & \Delta_{e\mu}^{++} & \Delta_{e\mu}^{+-} \\ \Delta_{ee}^{-+} & \Delta_{ee}^{--} & \Delta_{e\mu}^{-+} & \Delta_{e\mu}^{--} \\ \Delta_{\mu e}^{++} & \Delta_{\mu e}^{+-} & \Delta_{\mu\mu}^{++} & \Delta_{\mu\mu}^{+-} \\ \Delta_{\mu e}^{-+} & \Delta_{\mu e}^{--} & \Delta_{\mu\mu}^{-+} & \Delta_{\mu\mu}^{--} \end{pmatrix}, \tag{8.11}$$

where

$$\Delta_{kl}^{ss'} = \langle \nu_k^s | \Delta H_{SM} | \nu_l^{s'} \rangle, \qquad k,l = e,\mu;\ s,s' = \pm \tag{8.12}$$

$$\Delta H_{SM} = \frac{G_F}{2\sqrt{2}} n(1+\gamma_5)\vec{v}\vec{\gamma}, \tag{8.13}$$

$$\vec{v}\vec{\gamma} = v_1\gamma_1 + v_2\gamma_2 + v_3\gamma_3 \tag{8.14}$$

We have to introduce neutrino states as the superpositions of the mass states:

$$\nu_\alpha^s = C_\alpha\sqrt{\frac{E_\alpha + m_\alpha}{2E_\alpha}}\begin{pmatrix} u_\alpha^s \\ \dfrac{\vec{\sigma}\vec{p}_\alpha}{E_\alpha + m_\alpha} u_\alpha^s \end{pmatrix} e^{i\vec{p}_\alpha\vec{x}}, \alpha = 1,2, \tag{8.15}$$

$$u_\alpha^{s=1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{8.16}$$

$$u_\alpha^{s=-1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{8.17}$$

I would like to recall that we use the standard representation of γ −matrices:

$$\gamma_5 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \tag{8.18}$$

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{8.19}$$

$$\gamma_k = \begin{pmatrix} 0 & -\sigma_k \\ \sigma_k & 0 \end{pmatrix}, \tag{8.20}$$

$\sigma_k -$ Pauli-matrices. To evaluate for instance the term $\langle \nu_e^{\pm}|\Delta H_{SM}|\nu_e^{\pm}\rangle$ we first have to decompose the flavor states in terms of the mass states. So we should calculate the following term:

$$\langle \nu_1^s|\Delta H_{SM}|\nu_2^{s'}\rangle \sim \langle \bar{\nu}_1^s \vec{v}\vec{\gamma}(1+\gamma_5)\nu_2^{s'}\rangle = \left(\nu_1^{s+}\gamma_0(1+\gamma_5)\vec{v}\vec{\gamma}\nu_2^{s'}\right)$$
$$= -\nu_1^{s+}\begin{pmatrix} 0 & \vec{\sigma}\vec{v} \\ \vec{\sigma}\vec{v} & 0 \end{pmatrix}(1+\gamma_5) = \nu_1^{s+}\begin{pmatrix} \vec{\sigma}\vec{v} & \vec{\sigma}\vec{v} \\ \vec{\sigma}\vec{v} & \vec{\sigma}\vec{v} \end{pmatrix}\nu_2^{s'}, \tag{8.21}$$

where

$$\begin{pmatrix} \vec{\sigma}\vec{v} & \vec{\sigma}\vec{v} \\ \vec{\sigma}\vec{v} & \vec{\sigma}\vec{v} \end{pmatrix} = \begin{pmatrix} \vec{\sigma}\vec{v} & 0 \\ 0 & \vec{\sigma}\vec{v} \end{pmatrix} + \begin{pmatrix} 0 & \vec{\sigma}\vec{v} \\ \vec{\sigma}\vec{v} & 0 \end{pmatrix} = A + B \tag{8.22}$$

Firstly

$$A: \left(u_1^{s+}, \frac{\vec{\sigma}\vec{p}_1}{E_1+m_1}u_1^{s+}\right)\begin{pmatrix} \vec{\sigma}\vec{v} & 0 \\ 0 & \vec{\sigma}\vec{v} \end{pmatrix}\begin{pmatrix} u_2^{s'} \\ \frac{\vec{\sigma}\vec{p}_2}{E_2+m_2}u_2^{s'} \end{pmatrix}$$
$$= \left(u_1^{s+}\vec{\sigma}\vec{v}, \frac{\vec{\sigma}\vec{p}_1\vec{\sigma}\vec{v}}{E_1+m_1}u_1^{s+}\right)\begin{pmatrix} u_2^{s'} \\ \frac{\vec{\sigma}\vec{p}_2}{E_2+m_2}u_2^{s'} \end{pmatrix} \tag{8.23}$$

We should distribute the velocity vector between longitudinal and transversal components in respect to neutrino propagation:

$$\vec{v} = \vec{v}_{\parallel} + \vec{v}_{\perp} \tag{8.24}$$

We will use the frame shown on fig. 7.1 where $\vec{v}_{\perp} \,//\, \vec{i}$ and $\vec{v}_{\parallel} \,//\, \vec{k}$.

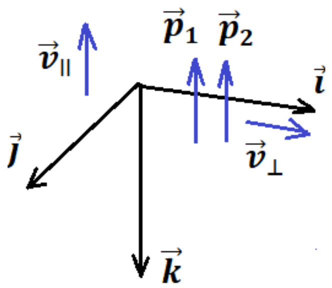


Fig. 7.1. Frame.

Continuing our calculations we shall use the well-known expression:

$$(\vec{\sigma}\vec{a})(\vec{\sigma}\vec{b}) = (\vec{a}\vec{b}) + i\vec{\sigma}[\vec{a}\vec{b}] \Rightarrow \tag{8.25}$$

$$(\vec{\sigma}\vec{p}_1)(\vec{\sigma}\vec{v})(\vec{\sigma}\vec{p}_2) = \{(\vec{p}_1\vec{v}) + i\vec{\sigma}[\vec{p}_1\vec{v}]\}(\vec{\sigma}\vec{p}_2)$$
$$= (\vec{p}_1\vec{v})(\vec{\sigma}\vec{p}_2) + i\{([\vec{p}_1\vec{v}]\vec{p}_2) - [[\vec{p}_1\vec{v}]\vec{p}_2]\vec{\sigma}\}, \tag{8.26}$$

where

$$([\vec{p}_1\vec{v}]\vec{p}_2) = 0, \tag{8.27}$$
$$(\vec{p}_1\vec{v})(\vec{\sigma}\vec{p}_2) = (p_1 v_{\parallel})(\sigma_3 p_2) = p_1 p_2 v_{\parallel}\sigma_3, \tag{8.28}$$
$$-[[\vec{p}_1\vec{v}]\vec{p}_2]\vec{\sigma} = \vec{\sigma}[\vec{p}_2[\vec{p}_1\vec{v}]] = \vec{\sigma}[\vec{p}_2[\vec{p}_1, \vec{v}_{\parallel} + \vec{v}_{\perp}]] = \vec{\sigma}[\vec{p}_2[\vec{p}_1, \vec{v}_{\perp}]]$$
$$= \vec{\sigma}\{\vec{p}_1(\vec{p}_2\vec{v}_{\perp}) - \vec{v}_{\perp}(\vec{p}_2\vec{p}_1)\} = -\vec{\sigma}_{\perp}\vec{v}_{\perp}(\vec{p}_2\vec{p}_1), \tag{8.29}$$
$$\vec{p}_1(\vec{p}_2\vec{v}_{\perp}) = 0, \tag{8.30}$$

$$\vec{\sigma}_\perp \vec{v}_\perp = \sigma_1 v_1 + \sigma_2 v_2 = \sigma_1 v_1 \tag{8.31}$$

Finally we get

$$(\vec{\sigma}\vec{p}_1)(\vec{\sigma}\vec{v})(\vec{\sigma}\vec{p}_2) = p_1 p_2 v_{||}\sigma_3 - p_1 p_2 v_1 \sigma_1 \tag{8.32}$$

Let's put (8.32) in (8.23):

$$A:\ u_1^{s+}\vec{\sigma}\vec{v}u_2^{s'} + u_1^{s+}\frac{\vec{\sigma}\vec{p}_1\vec{\sigma}\vec{v}\vec{\sigma}\vec{p}_2}{(E_1+m_1)(E_2+m_2)}u_2^{s'}$$
$$= u_1^{s+}(\sigma_3 v_{||} + \sigma_1 v_1)u_2^{s'} + u_1^{s+}\left\{\frac{p_1p_2v_{||}\sigma_3 - p_1p_2v_1}{(E_1+m_1)(E_2+m_2)}\sigma_1\right\}u_2^{s'}$$
$$= u_1^{s+}\left(\sigma_3 v_{||}\left[1+\frac{p_1p_2}{(E_1+m_1)(E_2+m_2)}\right] + \sigma_1 v_1\left[1-\frac{p_1p_2}{(E_1+m_1)(E_2+m_2)}\right]\right)u_2^{s'} \tag{8.33}$$

I would like to recall the general term that was in the expression for the exact solution for the mass states in vacuum:

$$\left\{1+\frac{p_1p_2}{(E_1+m_1)(E_2+m_2)}\right\}\sqrt{\frac{(E_1+m_1)(E_2+m_2)}{4E_1E_2}} \tag{8.34}$$

We should express everything in terms of

$$\gamma_1 = \frac{E_1}{m_1} \gg 1, \tag{8.35}$$

$$\gamma_2 = \frac{E_2}{m_2} \gg 1, \tag{8.36}$$

$$\gamma_{12}^{-1} = \frac{1}{2}(\gamma_2^{-1} + \gamma_2^{-1}), \gamma_{12} \gg 1 \Rightarrow \tag{8.37}$$

$$\sqrt{\frac{(E_1+m_1)(E_2+m_2)}{4E_1E_2}} \to \frac{1}{2}(1+\gamma_{12}^{-1}), \tag{8.38}$$

$$\frac{\vec{p}_\alpha}{E_\alpha + m_\alpha} \approx 1 - \gamma_\alpha^{-1} + o(\gamma_\alpha^{-2}) \tag{8.39}$$

Finally, for A term we get

$$A:\ u_1^{s+}(\sigma_3 v_{||} + \sigma_1 v_1 \gamma_{12}^{-1})u_2^{s'} \tag{8.40}$$

Secondly, we should calculate B term:

$$B:\ \left(u_1^{s+}, \frac{\vec{\sigma}\vec{p}_1}{E_1+m_1}u_1^{s+}\right)\begin{pmatrix}0 & \vec{\sigma}\vec{v}\\ \vec{\sigma}\vec{v} & 0\end{pmatrix}\begin{pmatrix}u_2^{s'}\\ \dfrac{\vec{\sigma}\vec{p}_2}{E_2+m_2}u_2^{s'}\end{pmatrix} = \cdots =$$

$$u_1^{s+}\left(v_{||} + i\sigma_2 v_\perp \frac{1}{2}\left(\frac{1}{\gamma_2} - \frac{1}{\gamma_1}\right)\right)u_2^{s'} \tag{8.41}$$

If we put together A and B terms we will get the final result:

$$u_1^{s+}\left\{(1-\sigma_3)v_{||} + \frac{1}{\gamma_{12}}\sigma_1 v_\perp + i\left(\frac{1}{\gamma_2} - \frac{1}{\gamma_1}\right)\sigma_2 v_\perp\right\}u_2^{s'}e^{i\vec{p}_\alpha\vec{x}} \tag{8.42}$$

Now we can write

$$\Delta_{\alpha\alpha'}^{ss'} = \frac{G_F}{2\sqrt{2}} \frac{n_0}{\sqrt{1-\vec{v}^2}} \left\{ u_\alpha^{s\,T} \left[(1-\sigma_3) v_{||} + \left(\gamma_{\alpha\alpha'}^{-1} \sigma_1 + i\tilde{\gamma}_{\alpha\alpha'}^{-1} \right) v_\perp \right] u_{\alpha'}^{s'} \right\} \delta_\alpha^{\alpha'}, \tag{8.43}$$

where

$$\tilde{\gamma}_{\alpha\alpha'}^{-1} = \frac{1}{2}\left(\gamma_\alpha^{-1} - \gamma_{\alpha'}^{-1}\right), \tag{8.44}$$

$$\gamma_{\alpha\alpha'}^{-1} = \frac{1}{2}\left(\gamma_\alpha^{-1} + \gamma_{\alpha'}^{-1}\right) \tag{8.45}$$

We recall the expression for Pauli-matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{8.46}$$

$$\sigma_2 = i\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{8.47}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{8.48}$$

Therefore

$$\boxed{\Delta_{\alpha\alpha'}^{ss'} = \frac{G_F}{2\sqrt{2}} \frac{n_0}{\sqrt{1-\vec{v}^2}} \left\{ u_\alpha^{s\,T} \left[\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} v_{||} + \begin{pmatrix} 0 & \gamma_\alpha^{-1} \\ \gamma_{\alpha'}^{-1} & 0 \end{pmatrix} v_\perp \right] u_{\alpha'}^{s'} \right\} \delta_\alpha^{\alpha'}} \tag{8.49}$$

This expression allows us to reconstruct all the elements of the addition to the effective evolution Hamiltonian for the flavor neutrinos due to matter motion.

By the way, we see that only term $\begin{pmatrix} 0 & \gamma_\alpha^{-1} \\ \gamma_{\alpha'}^{-1} & 0 \end{pmatrix} v_\perp$ in (8.49) determines the transition between neutrinos with opposite spin orientations $s \neq s'$.

Lecture 9. Neutrino spin-flavor oscillations in moving matter and constant magnetic field

The addition to the effective Hamiltonian due to moving matter

We continue considering the effect of spin and spin-flavor oscillations that appear due to weak interaction of neutrinos with the transversally moving matter. Last time we have derived the general expression for the contribution to the effective neutrino evolution Hamiltonian due to this effect. I would like to recall the neutrino evolution equation in the moving matter and external magnetic field $\vec{B} = \vec{B}_{\parallel} + \vec{B}_{\perp}$. We consider two different flavor states and two spin states:

$$\nu_f = \left(\nu_e^+, \nu_e^-, \nu_\mu^+, \nu_\mu^-\right)^T \tag{9.1}$$

The evolution equation looks like

$$i\frac{d}{dt}\nu_f = \left(H_{vac}^+ + H_B^f + H_{matt}^f\right)\nu_f, \tag{9.2}$$

where H_{vac}^+ describes neutrino evolution in vacuum, H_B^f accounts for the interaction of neutrino magnetic moment with external magnetic field in flavor basis, H_{matt}^f accounts for

1. matter at rest
2. longitudinal matter current component in respect to neutrino propagation – $\vec{j}_{\parallel}$
3. transversal matter current – $\vec{j}_{\perp}$

This matter term can be calculated in flavor basis as

$$H_{matt}^f = UH_vU^+, \tag{9.3}$$

where $v -$ the speed of matter. The term H_v has been calculated in mass basis:

$$H_v = H_{\substack{matter \\ at\ rest}} + \Delta H_{SM}, \tag{9.4}$$

where $\Delta H_{SM} -$ the contribution due to the moving matter:

$$\Delta H_{SM} = \begin{pmatrix} \Delta_{ee}^{++} & \Delta_{ee}^{+-} & \Delta_{e\mu}^{++} & \Delta_{e\mu}^{+-} \\ \Delta_{ee}^{-+} & \Delta_{ee}^{--} & \Delta_{e\mu}^{-+} & \Delta_{e\mu}^{--} \\ \Delta_{\mu e}^{++} & \Delta_{\mu e}^{+-} & \Delta_{\mu\mu}^{++} & \Delta_{\mu\mu}^{+-} \\ \Delta_{\mu e}^{-+} & \Delta_{\mu e}^{--} & \Delta_{\mu\mu}^{-+} & \Delta_{\mu\mu}^{--} \end{pmatrix}, \tag{9.5}$$

$$\Delta_{\alpha\alpha'}^{ss'} = \frac{G_F}{2\sqrt{2}}\frac{n_0}{\sqrt{1-\vec{v}^2}}\left\{u_\alpha^{s\,T}\left[\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}v_{\parallel} + \begin{pmatrix} 0 & \gamma_\alpha^{-1} \\ \gamma_{\alpha'}^{-1} & 0 \end{pmatrix}v_{\perp}\right]u_{\alpha'}^{s'}\right\}\delta_\alpha^{\alpha'}, \tag{9.6}$$

where $s, s' = \pm$; $\alpha, \alpha' = 1{,}2$. We supposed that matter is composed of neutrons. The term $\frac{n_0}{\sqrt{1-\vec{v}^2}}$ accounts for the Lorentz transformation from the laboratory frame to the frame related to matter at rest. The expression (9.6) shows that weak interaction of neutrino with transversally moving of matter generates the transitions between different spin states. This phenomenon for the first time was proposed in my paper in 2004 on the basis of quasi-classical approach to neutrino spin evolution using the Bargmann-Michele-Telegdi equation generalized for the case when matter is present. In 2018 I and my student Pustoshny have presented the quantum calculations for this effect.

The probability of spin and spin-flavor oscillations in moving matter and constant magnetic field

Let's calculate the probability of spin and spin-flavor oscillations in moving matter $\vec{j} = \vec{j}_{||} + \vec{j}_{\perp}$ and constant magnetic field $\vec{B} = \vec{B}_{||} + \vec{B}_{\perp}$.

1) Firstly, we need to make the transition to flavor basis:

$$\nu_f = U\nu_\alpha, \tag{9.7}$$

where $f = e, \mu$ and $\alpha = 1,2$. The mixing matrix is determined by mixing angle:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{9.8}$$

For the case of 4 neutrino spices: 2 flavor and 2 mass states we have

$$U = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & -\sin\theta & 0 & \cos\theta \end{pmatrix} \tag{9.9}$$

The evolution equation in flavor basis:

$$i\frac{d\nu_f}{dt} = H^f\nu_f, \tag{9.10}$$

$$H^f = UHU^+, \tag{9.11}$$

where $H^f -$ the Hamiltonian in flavor basis, $H -$ the Hamiltonian in mass basis. We can use the similarity between the transitions of Hamiltonians that describe the electromagnetic interaction with magnetic field and weak interaction with moving matter. We should compare two Hamiltonians. The Hamiltonian that describes the electromagnetic interaction in mass basis:

$$H_B = -\mu_{\alpha\alpha'}\bar{\nu}_{\alpha'}\vec{\Sigma}\vec{B}\nu_\alpha + h.c., \tag{9.12}$$

$$\vec{\Sigma}_i = \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} \tag{9.13}$$

The Hamiltonian that describes the weak interaction with moving matter in mass basis:

$$H_\nu = \tilde{G}_F n\bar{\nu}_{\alpha'}\vec{v}\vec{\gamma}\nu_\alpha, \tag{9.14}$$

$$\tilde{G}_F = \frac{G_F}{2\sqrt{2}}, \tag{9.15}$$

$$n = \frac{n_0}{\sqrt{1-\vec{v}^2}}, \tag{9.16}$$

$n_0 -$ invariant number density of neutrons. We know how the Hamiltonian (9.12) will be transformed by (9.11), we can apply the same transition to (9.14). I would like to repeat previous calculations for the magnetic moment interaction Hamiltonian:

$$H_B^f = UH_BU^+ = \begin{pmatrix} -\left(\frac{\mu}{\gamma}\right)_{ee} B_{\parallel} & \mu_{ee}B_{\perp} & -\left(\frac{\mu}{\gamma}\right)_{e\mu} B_{\parallel} & \mu_{e\mu}B_{\perp} \\ \mu_{ee}B_{\perp} & \left(\frac{\mu}{\gamma}\right)_{ee} B_{\parallel} & \mu_{e\mu}B_{\perp} & \left(\frac{\mu}{\gamma}\right)_{e\mu} B_{\parallel} \\ -\left(\frac{\mu}{\gamma}\right)_{e\mu} B_{\parallel} & \mu_{e\mu}B_{\perp} & -\left(\frac{\mu}{\gamma}\right)_{\mu\mu} B_{\parallel} & \mu_{\mu\mu}B_{\perp} \\ \mu_{e\mu}B_{\perp} & \left(\frac{\mu}{\gamma}\right)_{e\mu} B_{\parallel} & \mu_{\mu\mu}B_{\perp} & \left(\frac{\mu}{\gamma}\right)_{\mu\mu} B_{\parallel} \end{pmatrix}, \tag{9.17}$$

where $\mu_{ee}, \mu_{\mu\mu} -$ effective magnetic moments in flavor basis, $\mu_{e\mu} -$ transition magnetic moment in flavor basis:

$$\mu_{ee} = \mu_{11}\cos^2\theta + \mu_{22}\sin^2\theta + \mu_{12}\sin 2\theta, \tag{9.18}$$

$$\mu_{\mu\mu} = \mu_{11}\sin^2\theta + \mu_{22}\cos^2\theta - \mu_{12}\sin 2\theta, \tag{9.19}$$

$$\mu_{e\mu} = \mu_{12}\cos 2\theta + \frac{1}{2}(\mu_{22} - \mu_{11})\sin 2\theta, \tag{9.20}$$

$$\left(\frac{\mu}{\gamma}\right)_{ee} = \frac{\mu_{11}}{\gamma_{11}}\cos^2\theta + \frac{\mu_{22}}{\gamma_{22}}\sin^2\theta + \frac{\mu_{12}}{\gamma_{12}}\sin 2\theta, \tag{9.21}$$

$$\left(\frac{\mu}{\gamma}\right)_{\mu\mu} = \frac{\mu_{11}}{\gamma_{11}}\sin^2\theta + \frac{\mu_{22}}{\gamma_{22}}\cos^2\theta - \frac{\mu_{12}}{\gamma_{12}}\sin 2\theta, \tag{9.22}$$

$$\left(\frac{\mu}{\gamma}\right)_{e\mu} = \frac{\mu_{12}}{\gamma_{12}}\cos 2\theta + \frac{1}{2}\left(\frac{\mu_{22}}{\gamma_{22}} - \frac{\mu_{11}}{\gamma_{11}}\right)\sin 2\theta, \tag{9.23}$$

$$\gamma_1 = \frac{E_1}{m_1}, \tag{9.24}$$

$$\gamma_2 = \frac{E_2}{m_2}, \tag{9.25}$$

$$\gamma_{12}^{-1} = \frac{1}{2}(\gamma_2^{-1} + \gamma_2^{-1}) \tag{9.26}$$

We see that the term (9.23) relates to different flavor states, it is not zero if the transition magnetic moment in mass basis is not zero $\mu_{12} \neq 0$ or the magnetic moments for the mass states do not equal to each other $\mu_{11} \neq \mu_{22}$.

Now let's transform the Hamiltonian (9.14) that describes the weak interaction with the moving matter:

$$H_\nu^f = UH_\nu U^+ = \tilde{G}_F n \begin{pmatrix} 0 & -\left(\frac{\eta}{\gamma}\right)_{ee} v_\perp & 0 & \left(\frac{\eta}{\gamma}\right)_{e\mu} v_\perp \\ \left(\frac{\eta}{\gamma}\right)_{ee} v_\perp & 2\eta_{ee}(1-v_\parallel) & \left(\frac{\eta}{\gamma}\right)_{e\mu} v_\perp & \eta_{e\mu} \\ 0 & \left(\frac{\eta}{\gamma}\right)_{e\mu} v_\perp & 0 & \left(\frac{\eta}{\gamma}\right)_{\mu\mu} v_\perp \\ \left(\frac{\eta}{\gamma}\right)_{e\mu} v_\perp & \eta_{e\mu} & \left(\frac{\eta}{\gamma}\right)_{\mu\mu} v_\perp & \eta_{\mu\mu}(1-v_\parallel) \end{pmatrix}, \quad (9.27)$$

where

$$\eta_{ee} = \mu_{ee}\big|_{\substack{\mu_{11}=\mu_{22}=1,\\ \mu_{12}=0}} = 1, \quad (9.28)$$

$$\eta_{\mu\mu} = \mu_{\mu\mu}\big|_{\substack{\mu_{11}=\mu_{22}=1,\\ \mu_{12}=0}} = 1, \quad (9.29)$$

$$\eta_{e\mu} = \mu_{e\mu}\big|_{\substack{\mu_{11}=\mu_{22}=1,\\ \mu_{12}=0}} = 0, \quad (9.30)$$

$$\left(\frac{\eta}{\gamma}\right)_{ee} = \left(\frac{\mu}{\gamma}\right)_{ee}\bigg|_{\substack{\mu_{11}=\mu_{22}=1,\\ \mu_{12}=0}} = \frac{\cos^2\theta}{\gamma_1} + \frac{\sin^2\theta}{\gamma_2}, \quad (9.31)$$

$$\left(\frac{\eta}{\gamma}\right)_{\mu\mu} = \left(\frac{\mu}{\gamma}\right)_{\mu\mu}\bigg|_{\substack{\mu_{11}=\mu_{22}=1,\\ \mu_{12}=0}} = \frac{\sin^2\theta}{\gamma_1} + \frac{\cos^2\theta}{\gamma_2}, \quad (9.32)$$

$$\left(\frac{\eta}{\gamma}\right)_{e\mu} = \left(\frac{\mu}{\gamma}\right)_{e\mu}\bigg|_{\substack{\mu_{11}=\mu_{22}=1,\\ \mu_{12}=0}} = \frac{\sin 2\theta}{\gamma_{12}} \quad (9.33)$$

The terms $2\eta_{ee}(1-v_\parallel)$ and $\eta_{\mu\mu}(1-v_\parallel)$ correspond to matter at rest.

Particular case of spin oscillations

Let's consider the particular case of neutrino spin oscillations (without change of flavor) due to interaction with transversal magnetic field $B_\perp$ and weak interaction with transversally moving matter $j_\perp$:

$$\nu_e^L \leftarrow (B_\perp, j_\perp) \rightarrow \nu_e^R \quad (9.34)$$

From the general expression we can write

$$i\frac{d}{dt}\begin{pmatrix}\nu_e^L \\ \nu_e^R\end{pmatrix} = \begin{pmatrix} \left(\frac{\mu}{\gamma}\right)_{ee} B_\parallel + \eta_{ee}\tilde{G}_F n\big(1-\vec{v}\vec{\beta}\big) & \mu_{e\mu}B_\perp + \left(\frac{\eta}{\gamma}\right)_{ee} \tilde{G}_F n v_\perp \\ \mu_{ee}B_\perp + \left(\frac{\eta}{\gamma}\right)_{ee} \tilde{G}_F n v_\perp & -\left(\frac{\mu}{\gamma}\right)_{ee} B_\parallel - \eta_{ee}\tilde{G}_F n\big(1-\vec{v}\vec{\beta}\big) \end{pmatrix}\begin{pmatrix}\nu_e^L \\ \nu_e^R\end{pmatrix} \quad (9.35)$$

I would like to notice that I added the unix matrix multiplied by the constant value to the general expression to make it more symmetric. I also changed the definition of left and right-handed neutrinos because before we had

$$\nu_f = \left(\nu_e^+, \nu_e^-, \nu_\mu^+, \nu_\mu^-\right)^T \tag{9.36}$$

The term $\eta_{ee}\tilde{G}_F n\left(1 - \vec{v}\vec{\beta}\right)$ describes the matter at rest. We can see in (9.35) that even if we switch off the magnetic field there still will be mixing and oscillations between left and right-handed neutrinos due to weak interaction with transversal matter current. But this term $\left(\frac{\eta}{\gamma}\right)_{ee}\tilde{G}_F n v_\perp$ is suppressed by the big value of γ. In the case of magnetic field the longitudinal component $\left(\frac{\mu}{\gamma}\right)_{ee} B_{||}$ is suppressed. The longitudinal component of matter current is not suppressed but it is still very small because $\left|\vec{v}\vec{\beta}\right|$ is close to 1.

The probability of neutrino spin oscillations equals

$$P_{\nu_e^L \to \nu_e^R}(x) = \frac{E_{eff}^2}{E_{eff}^2 + \Delta_{eff}^2} \sin^2 \frac{\pi x}{L_{eff}}, \tag{9.37}$$

$$L_{eff} = \frac{\pi}{\sqrt{E_{eff}^2 + \Delta_{eff}^2}}, \tag{9.38}$$

$$E_{eff} = \left|\mu_{ee}\vec{B}_\perp + \left(\frac{\eta}{\gamma}\right)_{ee} \tilde{G}_F n \vec{v}_\perp\right|, \tag{9.39}$$

$$\Delta_{eff} = \left|\left(\frac{\mu}{\gamma}\right)_{ee} \vec{B}_{||} + \eta_{ee}\tilde{G}_F n\left(1 - \vec{v}\vec{\beta}\right)\vec{\beta}\right| \tag{9.40}$$

The resonance effect appears when the amplitude of oscillations is

$$\frac{E_{eff}^2}{E_{eff}^2 + \Delta_{eff}^2} = 1, \tag{9.41}$$

therefore, when Δ_{eff} is very small so

$$E_{eff}^2 \gg \Delta_{eff}^2 \tag{9.42}$$

Lecture 10. The probability of neutrino spin oscillations

Spin oscillations

We continue our discussion of the neutrino mixing and oscillations due to transversal matter currents. Last time we have derived the evolution Hamiltonian for 2 neutrino mass states, 2 neutrino flavor states and 2 spin states, so

$$\nu = \begin{pmatrix} \nu_e^L \\ \nu_e^R \\ \nu_\mu^L \\ \nu_\mu^R \end{pmatrix} \tag{10.1}$$

We have derived the complete evolution Hamiltonian in case of constant magnetic field $\vec{B} = \vec{B}_{||} + \vec{B}_\perp$ and moving matter composed of neutrons $\vec{J}_n = \vec{J}_{||} + \vec{J}_\perp$. In particular we considered the evolution of left and right electron neutrinos:

$$\nu = \begin{pmatrix} \nu_e^L \\ \nu_e^R \end{pmatrix} \tag{10.2}$$

We used the following notation for the neutrino spin oscillations (without change of flavor) due to interaction with transversal magnetic field $B_\perp$ and weak interaction with transversally moving matter $j_\perp$:

$$\nu_e^L \leftarrow (B_\perp, j_\perp) \rightarrow \nu_e^R \tag{10.3}$$

In our paper in 2018 I and my student Pustoshny also considered the transitions between different flavor states or spin-flavor oscillations:

$$\nu_e^L \leftarrow (B_\perp, j_\perp) \rightarrow \nu_\mu^R \tag{10.4}$$

We also included the effects of longitudinal motion of matter and longitudinal magnetic field. In the previous lecture we obtained the evolution equation for the case (10.3):

$$i\frac{d}{dt}\begin{pmatrix} \nu_e^L \\ \nu_e^R \end{pmatrix} = \begin{pmatrix} \left(\frac{\mu}{\gamma}\right)_{ee} B_{||} + \eta_{ee}\tilde{G}_F n\left(1 - \vec{v}\vec{\beta}\right) & \mu_{e\mu}B_\perp + \left(\frac{\eta}{\gamma}\right)_{ee}\tilde{G}_F n v_\perp \\ \mu_{ee}B_\perp + \left(\frac{\eta}{\gamma}\right)_{ee}\tilde{G}_F n v_\perp & -\left(\frac{\mu}{\gamma}\right)_{ee} B_{||} - \eta_{ee}\tilde{G}_F n\left(1 - \vec{v}\vec{\beta}\right) \end{pmatrix}\begin{pmatrix} \nu_e^L \\ \nu_e^R \end{pmatrix}, \tag{10.6}$$

where $\mu_{ee}, \mu_{\mu\mu}$ – effective magnetic moments and $\mu_{e\mu}$ – transition magnetic moment in flavor basis,

$$\mu_{ee} = \mu_{11}\cos^2\theta + \mu_{22}\sin^2\theta + \mu_{12}\sin 2\theta, \tag{10.7}$$

$$\eta_{ee} = 1, \tag{10.8}$$

$$\left(\frac{\eta}{\gamma}\right)_{ee} = \frac{\cos^2\theta}{\gamma_1} + \frac{\sin^2\theta}{\gamma_2}, \tag{10.9}$$

$$\gamma_i = \frac{E_i}{m_i} \tag{10.10}$$

The probability of spin oscillations

We would like to know when the transition $\nu_e^L \leftrightarrow \nu_e^R$ becomes important. The probability of neutrino spin oscillations can be calculated in the same way as the probability of neutrino flavor oscillations in magnetic field:

$$P_{\nu_e^L\to\nu_e^R}(x) = \frac{E_{eff}^2}{E_{eff}^2 + \Delta_{eff}^2}\sin^2\frac{\pi x}{L_{eff}}, \quad (10.11)$$

$$E_{eff} = \left|\mu_{ee}\vec{B}_\perp + \left(\frac{\eta}{\gamma}\right)_{ee}\tilde{G}_F n\vec{v}_\perp\right|, \quad (10.12)$$

$$\Delta_{eff} = \left|\left(\frac{\mu}{\gamma}\right)_{ee}\vec{B}_{\parallel} + \eta_{ee}\tilde{G}_F n\big(1-\vec{v}\vec{\beta}\big)\vec{\beta}\right| \quad (10.13)$$

The transition $\nu_e^L \leftrightarrow \nu_e^R$ becomes important or the resonance effect appears when the amplitude of oscillations is at maximum:

$$\frac{E_{eff}^2}{E_{eff}^2 + \Delta_{eff}^2} = 1, \quad (10.14)$$

therefore, when $\Delta_{eff}\to 0$ or at least

$$E_{eff} > \Delta_{eff} \quad (10.15)$$

It means that the amplitude of oscillations equals to $1/2$ or exceeds this value.

Firstly, let’s simplify the task and consider the case when magnetic field contribution is negligible. So now we are particularly interested in the effect of interaction with the moving matter. Therefore

$$E_{eff} = \left|\left(\frac{\eta}{\gamma}\right)_{ee}\tilde{G}_F n\vec{v}_\perp\right|, \quad (10.16)$$

$$\Delta_{eff} = \left|\eta_{ee}\tilde{G}_F n\big(1-\vec{v}\vec{\beta}\big)\vec{\beta}\right| \quad (10.17)$$

The oscillation length in (10.11) equals to

$$L_{eff} = \frac{\pi}{\sqrt{E_{eff}^2 + \Delta_{eff}^2}} = \frac{\pi}{\left(\frac{\eta}{\gamma}\right)_{ee}\tilde{G}_F n\vec{v}_\perp}, \quad (10.18)$$

$$\tilde{G}_F = \frac{G_F}{2\sqrt{2}} \quad (10.19)$$

From (10.15) it follows that

$$\left(\frac{\eta}{\gamma}\right)_{ee}\vec{v}_\perp \geq \big(1-\vec{v}\vec{\beta}\big) \quad (10.20)$$

For relativistic neutrinos $\beta \approx 1$. Let's suppose that the mass difference is much less than the absolute values:

$$\Delta m = m_2 - m_1 \ll m_1, m_2 \qquad (10.21)$$

Therefore

$$\frac{1}{\gamma_\nu} \sim \frac{1}{\gamma_1} \sim \frac{1}{\gamma_2} \Rightarrow \qquad (10.22)$$

$$\left(\frac{\eta}{\gamma}\right)_{ee} = \frac{\cos^2\theta}{\gamma_1} + \frac{\sin^2\theta}{\gamma_2} \sim \frac{1}{\gamma_\nu} \qquad (10.23)$$

From (10.23) and (10.20) we get the resonance condition:

$$\frac{v_\perp}{\gamma_\nu} \geq \left(1 - \vec{v}\vec{\beta}\right) \qquad (10.24)$$

As soon as $v_\perp < 1$

$$\frac{v_\perp}{\gamma_\nu} < \frac{1}{\gamma_\nu} \Rightarrow \qquad (10.25)$$

$$1 \gg \frac{1}{\gamma_\nu} \geq \left(1 - \vec{v}\vec{\beta}\right) \Rightarrow \qquad (10.26)$$

$$\left(1 - \vec{v}\vec{\beta}\right) \ll 1 \Rightarrow \qquad (10.27)$$

$$\cos\theta_{\vec{v}\vec{\beta}} > 0 \Rightarrow \qquad (10.28)$$

$$\left(1 - \vec{v}\vec{\beta}\right) > (1 - v\beta) \qquad (10.29)$$

Therefore, the resonance condition looks like

$$\frac{1}{\gamma_\nu} \geq (1 - v\beta), \qquad (10.30)$$

where $\frac{1}{\gamma_\nu} \ll 1$ and $(1 - v\beta) \approx (1 - v_{||})$, it means that

$$v_{||} \sim 1, v_\perp \ll v_{||} \qquad (10.31)$$

This result is not very good because as you remember

$$L_{eff} \sim \frac{1}{v_\perp} \qquad (10.32)$$

(In the presence of magnetic field we had $L_{eff} \sim \frac{1}{B_\perp}$.) It means that the dimension of space where this effect can appear is too big. Never the less, let's continue and introduce the γ −factor for the matter particles:

$$\gamma_n = \frac{1}{(1-\vec{v}^2)}, \tag{10.33}$$

where $v = v_n -$ the speed of neutrons:

$$v = \sqrt{1-\frac{1}{\gamma_n^2}} = 1 - \frac{1}{2}\frac{1}{\gamma_n^2} + \cdots \Rightarrow \tag{10.34}$$

$$\beta = 1 - \frac{1}{2}\frac{1}{\gamma_\nu^2} + \cdots \tag{10.35}$$

Then

$$(1-v\beta) = \left(1-\left(1-\frac{1}{2}\frac{1}{\gamma_n^2}+\cdots\right)\left(1-\frac{1}{2}\frac{1}{\gamma_\nu^2}+\cdots\right)\right) \approx \frac{1}{2}\left(\frac{1}{\gamma_n^2}+\frac{1}{\gamma_\nu^2}\right)\Big|_{\gamma_\nu \gg \gamma_n} \approx \frac{1}{2}\frac{1}{\gamma_n^2} \tag{10.36}$$

We consider that neutrinos are more relativistic than neutrons in matter. The resonance condition for the transition $\nu_e^L \overset{\vec{j}_\perp}{\leftrightarrow} \nu_e^R$:

$$\frac{1}{\gamma_\nu} \geq \frac{1}{2}\frac{1}{\gamma_n^2} \Rightarrow \tag{10.37}$$

$$\gamma_n > \sqrt{\frac{1}{2}\gamma_\nu} \tag{10.38}$$

When (10.38) is valid the amplitude of oscillations between left and right neutrinos produced by weak interaction with transversal matter current exceeds one half.

Now let’s consider the case when the masses equal to

$$m_1, m_2 \sim 0.1\ eV \tag{10.39}$$

and the energy of neutrino is

$$p_0^\nu \sim 10\ MeV \tag{10.40}$$

The condition (10.38) is valid:

$$\gamma_n \geq \gamma_n^{1/2} \sim 3 \cdot 10^3 \tag{10.41}$$

The effective oscillation length in this case:

$$L_{eff} = \frac{\pi}{\left(\frac{\eta}{\gamma}\right)_{ee}\tilde{G}_F n \vec{v}_\perp} = \frac{\pi\gamma_n}{\left(\frac{\eta}{\gamma}\right)_{ee}\tilde{G}_F n} = \cdots = \tag{10.42}$$

where

$$\left(\frac{\eta}{\gamma}\right)_{ee} \sim \frac{2}{\gamma_\nu}, \tag{10.43}$$

$$\tilde{G}_F n_0\Big|_{n_0\sim 10^{33}\ \text{см}^{-3}} = 10^{33}\ \text{см}^{-3}\,\frac{10^{-5}}{(1\ GeV)^2}\Bigg|_{\substack{(1\ \text{см})^{-1}=2\cdot 10^{-5}\ eV \\ 1\ eV=\frac{1}{2\cdot 10^{-5}}\ \text{см}}}$$

$$= 10^{33}\cdot 10^{-5}\cdot 8\cdot 10^{-15}(eV)^3\,\frac{1}{10^{18}(eV)^2} = 8\cdot 10^{-5}\ eV = \frac{4}{\text{см}}, \tag{10.44}$$

$c = \hbar = 1$. Returning to (10.42) we get that $L_{eff}\sim 50$ km for $n\sim 10^{37}$ and $\gamma_n \sim 3\cdot 10^3$. So, in order to have a sufficient amplitude of oscillations you need a huge density of transversal matter current, extremely relativistic neutrinos and a big scale where this effect should proceed.

Lecture 11. Introduction to neutrino electromagnetic properties

Articles

Firstly I would like to advertise my paper published together with my friend and collaborator Carlo Giunti from University of Torino in Italy. The title of this paper is "Neutrino electromagnetic interactions: A window to new physics". It was published in Reviews of Modern Physics in 2015. This paper collects mostly all important information about neutrino electromagnetic properties and neutrino electromagnetic interactions. Up to now about half a thousand citations of this paper could be found. Its ambitious title came from my early paper published in 2009 "Neutrino magnetic moment: a window to new physics". If you are interested in more detailed discussion on different aspects of neutrino electromagnetic properties I advise you to read this paper. When we've finished the paper the number of pages was far above the accepted limit and the editors suggested us to have a regular paper and some supplemental materials that also would be interesting for students because it contains many technical details and calculations.

There was another paper "Electromagnetic Properties of Neutrino" published in 2012 by Broggini, Giunti and me. It also devoted to neutrino electromagnetic properties and it's available on the Internet.

Also I would like to advertise very important fundamental textbook of a famous German scientist Georg G. Raffelt. The title of this book is "Stars as Laboratories for Fundamental Physics". Indeed, when we discuss neutrino electromagnetic properties we always think about the possibility to observe the manifestation of non-trivial neutrino electromagnetic properties in astrophysics.

Historical introduction

I would like to recall some historical issues related to the appearance of neutrino that we have already discussed. Pauli proposed the existence of neutrino in 1930. He called this particle "neutron". Three years later in 1933 E. Fermi used this idea of existence of neutrino and developed a new model of connection between different particles that we now call the theory of weak interactions. He renamed Pauli's neutron to neutrino. Pauli also supposed that neutrino probably had non-trivial electromagnetic properties and magnetic moment of neutrino could be not zero $\mu_\nu \neq 0$.

There is the famous statement of Pauli: "Today I did something a physicist should never do. I predicted something which will never be observed experimentally…" For at least three decades my group has been studying neutrino electromagnetic properties, sometimes our colleagues saying to us that we are studying something that is nearly invisible or the effects that are very hard to observe and I always address them to this Pauli's statement. Although all the properties of this particle are very hard to observe neutrino plays a crucial role in the structure of the universe.

Another Nobel Prize winner H. Bethe with his co-author R. Peierls in the very ambitious paper “The neutrino” published in 1934 in Nature stated: “There is no practically possible way of observing the neutrino.” Indeed neutrino always brings us puzzles. We know that the mass of neutrino is not zero $m_\nu \neq 0$ but what is the absolute value?

Neutrino puzzles

It is very interesting to investigate astrophysical applications of neutrino electromagnetic properties because, indeed, neutrino manifests itself very clearly under the influence of extreme external conditions that can be found in astrophysics where very strong external electromagnetic fields or very dense background matter are present.

To say the truth electromagnetic properties are really a puzzle. In spite of great efforts the results of terrestrial laboratory experiments as well as data from astrophysical and cosmology observations are in perfect agreement with the hypothesis of zero neutrino electromagnetic properties. However we know from the discovery of neutrino oscillations that neutrino mass is not zero inevitably it means that neutrino magnetic moment should be non-zero. The Dirac neutrino magnetic moment equals

$$\mu_{ii}^D = \frac{3eG_F m_i}{8\sqrt{2}\pi^2} \approx 3.2 \times 10^{-19} \left(\frac{m_i}{1\ eV}\right) \mu_B \tag{11.1}$$

This is a result of the one loop calculation in the easiest generalization of the Standard Model when the effect of neutrino mass is included. If we scale the mass of neutrino m_i on the value of 1 eV then

$$\mu_{ii}^D \sim 3.2 \times 10^{-19} \mu_B \tag{11.2}$$

From KATRIN experiment we know that the upper bound on the neutrino mass is $0{,}8\ eV$. This result was obtained in calculations performed by K. Fujikawa and R. Shrock in 1980. I would like to inform you that the value (11.2) is many orders of magnitude smaller than the existing bounds on magnetic moment from the present experimental investigations. GEMMA experiment obtained in 2012 gives us the following upper bound for the magnetic moment of neutrino:

$$\mu_\nu \sim 10^{-11} \mu_B \tag{11.3}$$

A little bit more strong constraint on neutrino magnetic moment comes from astrophysics from the studies of solar or supernova neutrino fluxes:

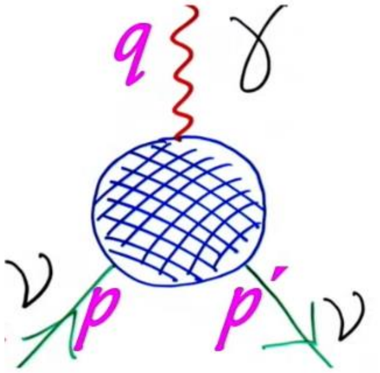
$$\mu_\nu \sim 10^{-11} \div 10^{-12} \mu_B \tag{11.4}$$

There are some generalizations of the Standard Model that do not forbid that neutrino is a millicharged particle. This value is also constrained and the most severe bound of the neutrino millicharge comes from neutrality of the hydrogen atom:

$$q_\nu \leq 3 \times 10^{-21} e \tag{11.5}$$

Astrophysical constraints are much weaker.

Neutrino electromagnetic properties theory



Pic. 11.1.

The neutrino electromagnetic interactions are described by the so-called electromagnetic vertex function $\Lambda_\mu(q,l)$. The diagram on pic. 11.1 shows us some interaction that connects initial and final neutrino states with the real photon. The matrix element of electromagnetic current is connected with electromagnetic vertex function:

$$\langle\psi(p')|J_\mu^{EM}|\psi(p)\rangle = \bar{u}(p')\Lambda_\mu(q,l)u(p) \qquad (11.6)$$

From some general prescriptions we can investigate the structure of this electromagnetic vertex function of neutrinos. In particular if we impose constraints from Lorentz covariance and electromagnetic gauge invariance we can obtain the exact expression of the electromagnetic vertex function which should be constructed using several matrices: $\hat{I}, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu}$, several tensors: $g_{\mu\nu}, \epsilon_{\mu\nu\sigma\gamma}$ and also two independent vectors q_μ and l_μ:

$$q_\mu = p'_\mu - p_\mu, \qquad (11.7)$$

$$l_\mu = p'_\mu + p_\mu \qquad (11.8)$$

We can write three sets of operators from which we can construct the vertex function in general form:

- $\hat{I}q_\mu, \hat{I}l_\mu, \gamma_5 q_\mu, \gamma_5 l_\mu, \not{q}q_\mu, \not{l}q_\mu, \gamma_5 q_\mu, \gamma_5\not{q}q_\mu, \gamma_5\not{l}q_\mu, \sigma_{\alpha\beta}q^\alpha l^\beta q_\mu, (q_\mu \leftrightarrow l_\mu)$
- $\gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu}q^\nu, \sigma_{\mu\nu}l^\nu$
- $\epsilon_{\mu\nu\sigma\gamma}\sigma^{\alpha\beta}q^\nu, \epsilon_{\mu\nu\sigma\gamma}\sigma^{\alpha\beta}l^\nu, \epsilon_{\mu\nu\sigma\gamma}\sigma^{\nu\beta}q_\beta q^\sigma l^\gamma, \epsilon_{\mu\nu\sigma\gamma}\sigma^{\nu\beta}l_\beta q^\sigma l^\gamma, \epsilon_{\mu\nu\sigma\gamma}\gamma^\nu q^\sigma l^\gamma \hat{I}, \epsilon_{\mu\nu\sigma\gamma}\gamma^\nu q^\sigma l^\gamma \gamma_5$

Using Gordon-like identities we get

$$\Lambda_\mu(q,l) = f_1(q^2)q_\mu + f_2(q^2)q_\mu\gamma_5 + f_3(q^2)\gamma_\mu + f_4(q^2)\gamma_\mu\gamma_5 + f_5(q^2)\sigma_{\mu\nu}q^\nu + f_6(q^2)\epsilon_{\mu\nu\rho\gamma}\sigma^{\rho\gamma}q^\nu, \qquad (11.9)$$

where the only dependence on q^2 remains because

$$p^2 = p'^2 = m^2, \qquad (11.10)$$

$$l^2 = 4m^2 - q^2 \qquad (11.11)$$

$$\begin{aligned}
\bar{u}(\mathbf{p}_1)\gamma^{\mu}u(\mathbf{p}_2) &= \frac{1}{2m}\bar{u}(\mathbf{p}_1)[l^{\mu} + i\sigma^{\mu\nu}q_{\nu}]u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)\gamma^{\mu}\gamma_5 u(\mathbf{p}_2) &= \frac{1}{2m}\bar{u}(\mathbf{p}_1)[\gamma_5 q^{\mu} + i\gamma_5\sigma^{\mu\nu}l_{\nu}]u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)i\sigma^{\mu\nu}l_{\nu}u(\mathbf{p}_2) &= -\bar{u}(\mathbf{p}_1)q^{\nu}u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)i\sigma^{\mu\nu}q_{\nu}u(\mathbf{p}_2) &= \bar{u}(\mathbf{p}_1)[2m\gamma^{\mu}l^{\mu}]u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)i\sigma^{\mu\nu}\gamma_5 q_{\nu}u(\mathbf{p}_2) &= -\bar{u}(\mathbf{p}_1)l^{\mu}\gamma_5 u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)[\epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_{\beta}q_{\mu}l_{\nu}]u(\mathbf{p}_2) &= \bar{u}(\mathbf{p}_1)\{-i[q^{\alpha}\ \not{l} - l^{\alpha}\ \not{q}] + i(q^2 - 4m^2)\gamma^{\alpha} + \\
&\quad 2im(l^{\alpha} + q^{\alpha})\}u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)[\epsilon^{\alpha\mu\nu\beta}\gamma_{\beta}q_{\mu}l_{\nu}]u(\mathbf{p}_2) &= \bar{u}(\mathbf{p}_1)\{i[q^{\alpha}\ \not{l} - l^{\alpha}\ \not{q}]\gamma_5 + iq^2\gamma_5\gamma^{\alpha} - \\
&\quad 2im(l^{\alpha} + q^{\alpha})\gamma_5\}u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)[\epsilon^{\mu\nu\alpha\beta}q_{\alpha}l_{\beta}\gamma_{\nu}\gamma_5]u(\mathbf{p}_2) &= \frac{i}{2m}\bar{u}(\mathbf{p}_1)[\epsilon^{\mu\nu\alpha\beta}q_{\alpha}l_{\beta}\sigma_{\nu\rho}q^{\rho}]u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)[\epsilon^{\mu\nu\alpha\beta}q_{\alpha}l_{\beta}\sigma_{\nu\rho}l^{\rho}]u(\mathbf{p}_2) &= 0
\end{aligned}$$

Pic. 11.2. Gordon-like identities.

We forgot to mention that there is a requirement of current conservation in terms of electromagnetic gauge invariance:

$$\partial_{\mu}j^{\mu} = 0 \tag{11.12}$$

Accounting for (11.12) we come to two relations between terms of the previous expression for the vertex function (11.9):

$$f_1(q^2)q^2 + f_2(q^2)q^2\gamma_5 + 2mf_4(q^2)\gamma_5 = 0 \Rightarrow \tag{11.13}$$

$$f_1(q^2) = 0, \tag{11.14}$$

$$f_2(q^2)q^2\gamma_5 + 2mf_4(q^2)\gamma_5 = 0 \tag{11.15}$$

Finally we come to very indeed modern independent expression form of the electromagnetic vertex function of neutrinos that is composed of four from factors:

$$\begin{aligned}\Lambda_{\mu}(q,l) = f_Q(q^2)\gamma_{\mu} + f_M(q^2)i\sigma_{\mu\nu}q^{\nu} \\ +f_E(q^2)\sigma_{\mu\nu}q^{\nu}\gamma_5 + f_A(q^2)\big(q^2\gamma_{\mu} - q_{\mu}/q\big)\gamma_5,\end{aligned} \tag{11.16}$$

where $f_Q -$ charge from factor, $f_M -$ magnetic from factor, $f_E -$ dipole electric from factor and $f_A -$ anapole from factor.

If we go further and also account for the hermiticity and discrete symmetries of electromagnetic current J_{μ}^{EM} we will obtain additional constraints on the form factors. On this stage we observe that Dirac and Majorana neutrinos exhibit quite different electromagnetic properties.

Dirac neutrino:

1) CP invariance + Hermiticity $\Rightarrow f_E = 0$

2) at zero momentum transfer only electric charge $f_Q(0)$ and magnetic moment $f_M(0)$ contribute to $H_{int} \sim J_\mu^{EM} A^\mu$
3) Hermiticity itself ⇒ three form factors are real: $Imf_Q = Imf_M = Imf_A = 0$

Majorana neutrino:

1) CPT invariance ⇒ $f_Q = f_M = f_E = 0$

We can say that the studies of neutrino electromagnetic properties provide a way for distinguishing the nature of neutrinos: whether neutrino is Dirac or Majorana particle. It is important to mention that as early as in 1939 Pauli stated that for Majorana neutrinos if CPT invariance is valid the three from factors should be zero:

$$f_Q = f_M = f_E = 0 \tag{11.17}$$

If we talk about some general theoretical framework it may happen that the matrix element of electromagnetic current J_μ^{EM} is considered between different initial $\psi_i(p)$ and final $\psi_j(p')$ neutrino mass states:

$$\langle \psi_j(p') \big| J_\mu^{EM} | \psi_i(p) \rangle = \bar{u}_j(p') \Lambda_\mu(q,l) u_i(p) \tag{11.18}$$

In this case

$$\Lambda_\mu(q) = \left(f_Q(q^2)_{ij} + f_A(q^2)_{ij}\right)\left(q^2\gamma_\mu - q_\mu/q\right) \\ + f_M(q^2)_{ij} i\sigma_{\mu\nu}q^\nu + f_E(q^2)_{ij}\sigma_{\mu\nu}q^\nu\gamma_5, \tag{11.19}$$

where the form factors are matrices in neutrino mass eigenstates space. There are diagonal and off-diagonal form factors. Dirac and Majorana neutrinos again exhibit quite different electromagnetic properties. For instance, the diagonal magnetic moments of Majorana neutrinos should be zero however Majorana neutrinos can have non-diagonal magnetic moments when due to electromagnetic interaction the different mass states of neutrino are coupled to each other. Once again the studies of neutrino electromagnetic properties provide a tool for distinguishing the nature of neutrinos. If diagonal magnetic moment is not zero $\mu_\nu \neq 0$ then it is Dirac particle; if non-diagonal or transitional magnetic moment is not zero $\mu_\nu \neq 0$ then it is Majorana particle.

We have discussed electromagnetic properties in case of real neutrino which is a massive particle but we also can describe electromagnetic properties in case of massless neutrino $m_\nu = 0$. For massless left-handed neutrino

$$\bar{u}(p')\Lambda_\mu(q)u(p) = f_D(q^2)\bar{u}(p')\gamma_\mu(1+\gamma_5)u(p) \tag{11.20}$$

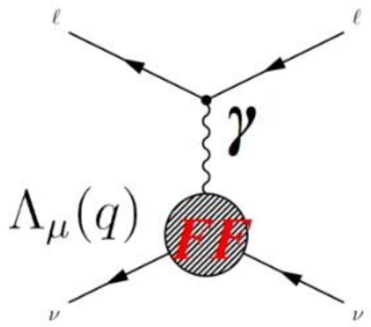
It's easy to show that in this case only two form factors describe neutrino electromagnetic interactions. The electric charge form factor f_Q and the anapole form factor f_A are related to each other and to the so-called Dirac form factor:

$$f_Q(q^2) = f_D(q^2), \tag{11.21}$$

$$f_A(q^2) = f_D(q^2)/q^2 \tag{11.22}$$

In gauge model the form factors at zero momentum transfer when $q^2 = 0$ are elements of the scatting matrix. In any consistent theoretical model these form factors provide determine static properties of neutrino that should be measured in direct interaction with external electromagnetic fields. This is realized for three of four form factors $f_Q(q^2)$, $f_M(q^2)$ and $f_E(q^2)$ in minimally extended Standard Model.

In non-Abellan gauge models the from factors at $q^2 = 0$ can be not invariant under gauge transformation because in general off-shell photon propagator is gauge dependent (pic. 11.3).



Pic. 11.3.

In this case the form factors in matrix element cannot be directly measured in experiment with interaction of neutrino with external electromagnetic field. We should consider the processes of higher order to reach the accessibility of electromagnetic characteristics for experimental observation.

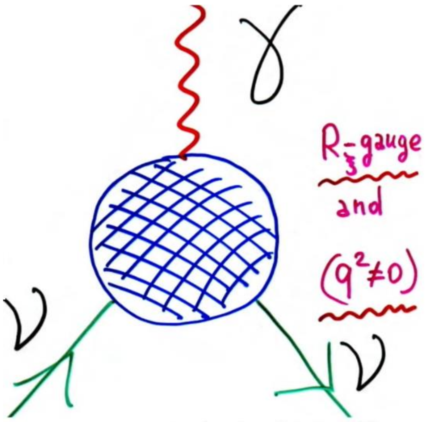
I would like to say that magnetic moment $\mu_\nu = f_M(0)$ is the most well studied and theoretically understood among neutrino electromagnetic properties.

Studenikin-Dvornikov research

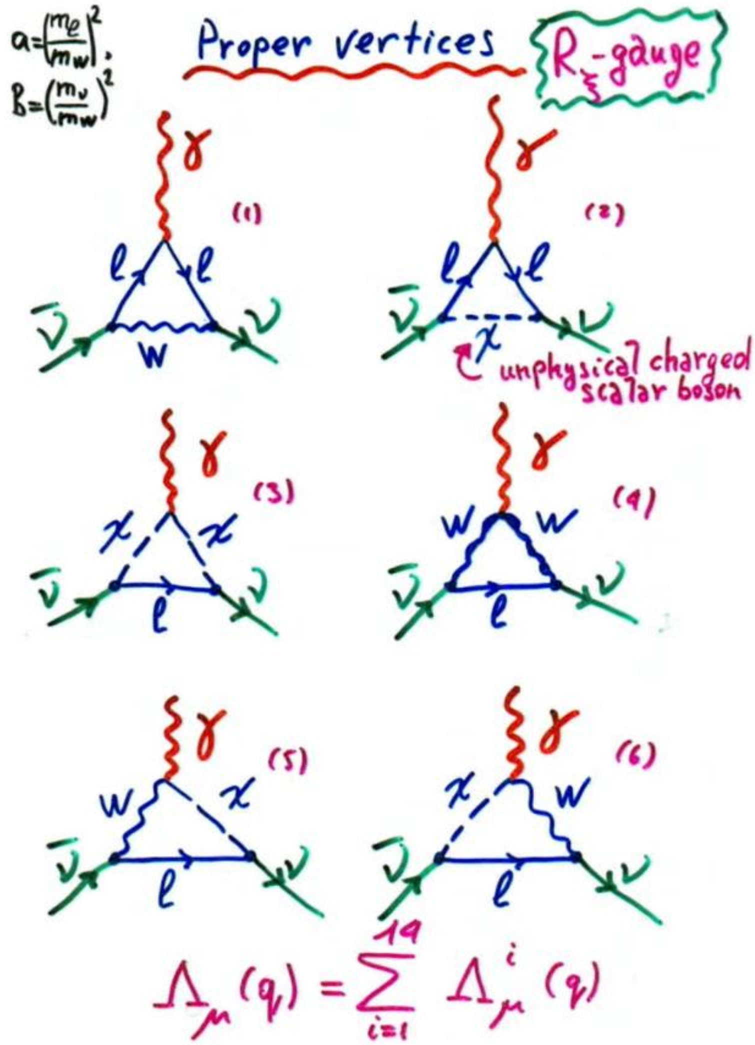
I and my student M. Dvornikov have performed the most general study of the massive neutrino electromagnetic vertex function (including electric and magnetic form factors) in arbitrary R_ξ gauge in the context of the SM + SU(2)-singlet ν_R also accounting for the masses of all particles in the polarization loops. There are there are two our papers: «Electric charge and magnetic moment of massive neutrino» published in Phys. Rer. D. in 2004 and «Electromagnetic form factors of a massive neutrino» published in JETP in 2004. We've calculated all one loop contributions to the diagram on pic. 11.4:

$$\Lambda_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2) i\sigma_{\mu\nu}q^\nu - \\ - f_E(q^2) i\sigma_{\mu\nu}q^\nu\gamma_5 - f_A(q^2)\left(q^2\gamma_\mu - q_\mu /q\right)\gamma_5 \qquad (11.23)$$

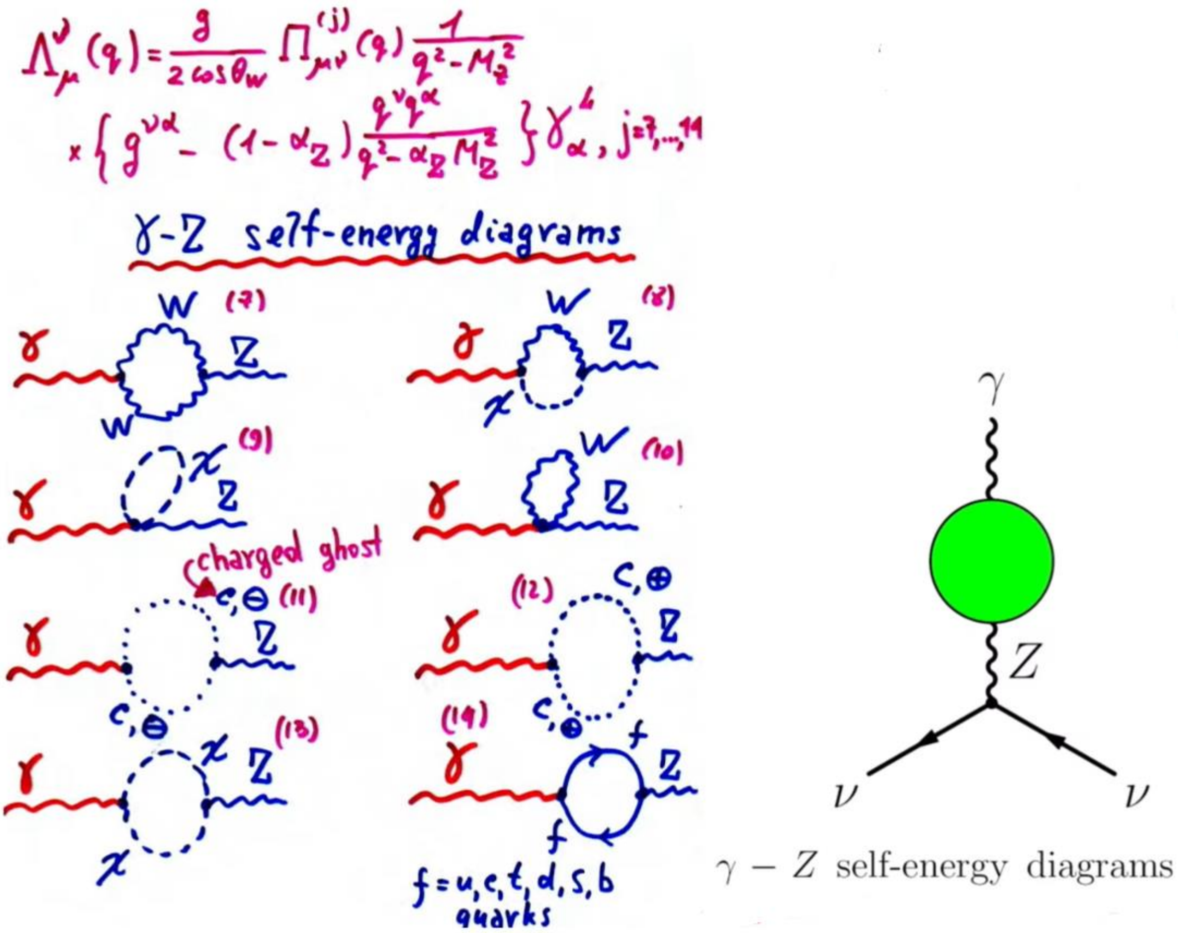
As I've mentioned we used the R_ξ-gauge and studied the dependence of form factors on $q^2 \neq 0$. The list of diagrams that have been calculated by us is shown on pic. 11.5 and 11.6.



Pic. 11.4.



Pic. 11.5. Proper vertices.



Pic. 11.6. $\gamma - Z$ self-energy diagrams.

Finally we obtained the exact expressions for four form factors and we also have proven that in the easiest generalization of the Standard Model the electric charge is exactly equal to zero $f_Q(0) = 0$ and gauge-independent. We also investigated the behavior of magnetic moment $f_M(0)$ and have found that it's finite and also gauge-independent.

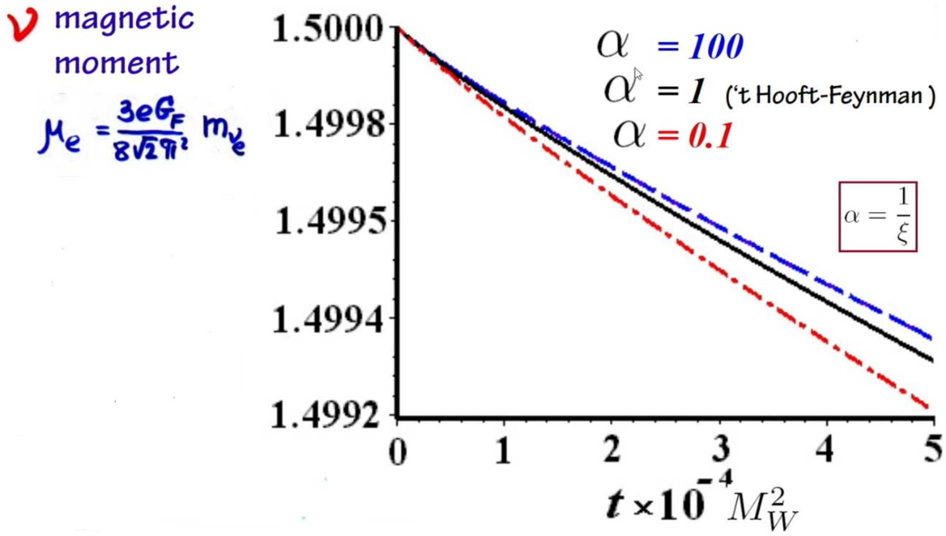
These studies enable us to investigate the dependence of neutrino magnetic moment on the mass of neutrino. We obtain the expression for neutrino magnetic moment $\mu(a, b, \alpha) = f_M(q^2 = 0)$ as a function of two mass parameters:

$$a = \left(\frac{m_e}{M_W}\right)^2, \qquad (11.24)$$

$$b = \left(\frac{m_\nu}{M_W}\right)^2, \qquad (11.25)$$

$$\mu(a, b, \alpha) = \sum_{i=1}^{6} \mu^{(i)}(a, b, \alpha) \qquad (11.26)$$

We also investigate the dependence of neutrino magnetic moment on the gauge-fixing parameter $\alpha = 1/\xi$. The results of our calculations presented on pic. 11.7. The vertical Y-axis indicates the value of magnetic moment and the X-axis indicates the q^2 value. We have three lines for three different gauge-fixing parameters α. For example, $\alpha = 1$ corresponds to the Hooft-Feynman gauge.



Pic. 11.7. The dependence of neutrino magnetic moment μ_ν on q^2.

We see that when q^2 tends to zero the magnetic moment calculated with three different values of α tends to one unique gauge-independent value.

These studies enable us to get an exact expression for magnetic moment for different ordering of neutrino masses:

- light neutrino $m_\nu \ll m_e \ll M_W$

$$\mu_\nu = \frac{eG_F}{4\pi^2\sqrt{2}} m_\nu \frac{3}{4(1-a)^3}(2 - 7a + 6a^2 - 2a^2 \ln a - a^3) \qquad (11.27)$$

- intermediate neutrino $m_e \ll m_\nu \ll M_W$

$$\mu_\nu = \frac{3eG_F}{8\pi^2\sqrt{2}} m_\nu \left\{1 + \frac{5}{18} b\right\} \qquad (11.28)$$

- heavy neutrino $m_e \ll M_W \ll m_\nu$

$$\mu_\nu = \frac{eG_F}{8\pi^2\sqrt{2}} m_\nu \qquad (11.29)$$

The next step should be including the mixing between different neutrinos. The results of calculations for transition neutrino magnetic and electric dipole moments:

$$\left.\begin{matrix}\mu_{ij}\\ \epsilon_{ij}\end{matrix}\right\} = \frac{3eG_F m_i}{32\sqrt{2}\pi^2}\left(1 \pm \frac{m_j}{m_i}\right)\left(\frac{m_\tau}{M_W}\right)^2 \sum_{l=e,\mu,\tau}\left(\frac{m_l}{m_\tau}\right)^2 U_{lj}U_{li}^*, \qquad (11.30)$$

$$\left.\begin{matrix}\mu_{ij}\\ \epsilon_{ij}\end{matrix}\right\} = 4 \cdot 10^{-23}\mu_B \left(\frac{m_i \pm m_j}{1\ eV}\right) \sum_{l=e,\mu,\tau}\left(\frac{m_l}{m_\tau}\right)^2 U_{lj}U_{li}^* \qquad (11.31)$$

We can see that these values are very strongly constrained due to so-called GIM cancellation mechanism according to which

$$f_{r_l} \to \cancel{-\frac{3}{2}} + \frac{3}{4}\left(\frac{m_l}{M_W}\right)^2 \ll 1 \tag{11.32}$$

This is by the way the reason why neutrino radioactive decay is very slow. The diagonal magnetic moment for Dirac neutrino:

$$\mu_{ii} = \frac{3eG_F m_i}{8\sqrt{2}\pi^2}\left(1 - \frac{1}{2}\sum_{l=e,\mu,\tau} r_l |U_{li}|^2\right) \approx 3.2 \cdot 10^{-19}\left(\frac{m_i}{1\ eV}\right)\mu_B, \tag{11.33}$$

$$r_l = \left(\frac{m_l}{M_W}\right)^2 \tag{11.34}$$

Other studies

Theorists try to construct a model where magnetic moment of neutrino will get much bigger value than predicted in the easiest generalization of the Standard Model and still acceptable mass. P. Vogel in 2006 stated that there is some new physics beyond the Standard Model that is characterized by some energy scale Λ and it provides an additional contribution to neutrino magnetic moment

$$\mu_\nu \sim \frac{eG}{\Lambda}, \tag{11.35}$$

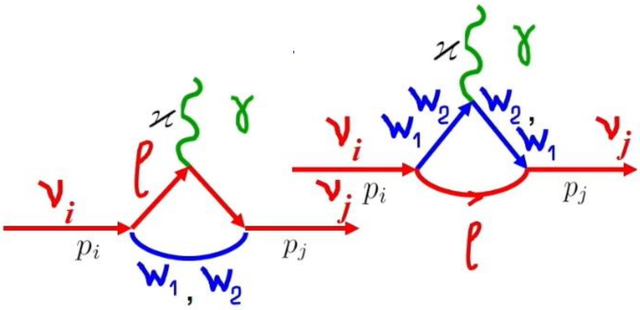
where $G -$ combination of some constants and loop factors. However we shouldn't forget that the mass of neutrino will also be shifted:

$$m_\nu \sim G\Lambda \tag{11.36}$$

Combining these two contributions we can obtain that

$$m_\nu \sim \frac{\Lambda^2}{2m_e}\frac{\mu_\nu}{\mu_B} \sim \frac{\mu_\nu}{10^{-18}\mu_B}[\Lambda(TeV)]^2\ eV \tag{11.37}$$

If we measure Λ in TeV range then any excess in neutrino magnetic moment contribution μ_ν larger than $10^{-18}\mu_B$ provided by the easiest generalization of the Standard Model gives an unacceptable big contribution to the mass of neutrino m_ν. Now we know that the mass of neutrino is constrained by KATRIN experiment on the level of $\sim 1eV$. The final conclusion from this discussion is that it's very hard to construct a theory which would provide a large neutrino magnetic moment μ_ν (much bigger than in the easiest generalization of the Standard Model) and at the same time remain an acceptable value of neutrino mass m_ν.



Pic. 11.8. Left-right symmetric model's diagrams.

There are several theoretical approaches by Kim (1976); Marciano, Sanda (1977); Beg, Marciano, Ruderman (1978) considering neutrino magnetic moment in so-called left-right symmetric models in which additional bosons are introduced (in respect to the Standard Model bosons). In such theoretical models it is possible that contribution to neutrino magnetic moment is not exactly proportional to the mass of neutrino m_{ν_l}, it has an additional term that is proportional to the mass of charged lepton m_l:

$$\mu_{\nu_l} = \frac{eG_F}{2\sqrt{2}\pi^2}\left[m_l\left(1 - \frac{m_{W_1}^2}{m_{W_2}^2}\right)\sin 2\xi + \frac{3}{4}m_{\nu_l}\left(1 + \frac{m_{W_1}^2}{m_{W_2}^2}\right)\right] \tag{11.38}$$

There are some theoretical models beyond the Standard Model that can lift neutrino magnetic moment to the level of present experimental constraints from both terrestrial laboratories and astrophysical and cosmology observations.

Electric charge of neutrino

You will probably be very much surprised but it is not completely excluded that the electric charge of neutrino is not zero. In the easiest generalization of the Standard Model the electrical neutrality of neutrino is attributed to the gauge invariance and the anomaly cancellation constraints. There are some models beyond the Standard Model where it is possible to overcome these constraints and consider neutrino as charged particle. However we know from experimental limits that the charge is quantized. Therefore these millicharged neutrinos should be very tiny.

Charge radius of neutrino

There is another very important electromagnetic characteristic of neutrino that indeed is not zero even in the Standard Model. Even if the electric charge of neutrino is vanishing, the electric form factor $f_Q(q^2)$ can still contain nontrivial information about neutrino static properties. A neutral particle can be characterized by a superposition of two charge distributions of opposite signs so that the particle's form factor $f_Q(q^2)$ can be non zero for $q^2 \neq 0$. In the case of an electrically neutral neutrino one usually introduces the so-called mean charge radius $\langle r_\nu^2 \rangle$ which is determined by the second term in the expansion of neutrino charge form factor $f_Q(q^2)$ in series powers of q^2:

$$f_Q(q^2) = f_Q(0) + q^2 \left.\frac{df_Q(q^2)}{dq^2}\right|_{q^2=0} + \cdots \Rightarrow \tag{11.39}$$

$$\langle r_\nu^2 \rangle = -6 \left.\frac{df_Q(q^2)}{dq^2}\right|_{q^2=0} \tag{11.40}$$

To understand what the charge radius of neutrino is, we can put the analogy with the elastic electron scattering on a static spherically symmetric charged distribution of some density $\rho(r)$. The differential cross section is determined by the corresponded form factor $f_Q(q^2)$:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}\bigg|_{point} \left|f_Q(q^2)\right|^2 \tag{11.41}$$

Continuing the calculations we can easily obtain the resulting formula (11.40). This value can be manifested in experiments of neutrino scattering on a target. By the way, the present theoretical prediction of neutrino charge radius is only one order of magnitude less than the sensitivity of measurement of the scattering of neutrino on electrons. It means that it's probably the most accessible electromagnetic property of neutrino for experimental observation.

I would like to mention that it is quite tricky correspondence between the charge radius and the fourth anapole form factor. To be correct in the Standard Model the charge radius and the anapole moment are not defined separately. Experiments are sensitive to a combination between the charge radius and the anapole moment:

$$f^{SM}(q^2) = \tilde{f}_Q(q^2) - f_A(q^2) \tag{11.42}$$

In the scattering experiment there is no direct information of each term individually.

Conclusion

So once again in the most general approach the electromagnetic vertex function of neutrinos is decomposed in terms of four form factors: electric, magnetic, electric dipole and anopole.

$$\Lambda_\mu^{if}(q) = f_Q^{if}(q^2)\gamma_\mu + f_M^{if}(q^2)i\sigma_{\mu\nu}q^\nu$$
$$+f_E^{if}(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A^{if}(q^2)\left(q^2\gamma_\mu - q_\mu/q\right)\gamma_5 \tag{11.43}$$

In case of $q^2 = 0$ the form factors provide the physical electromagnetic properties of neutrinos that can be measured in experiments.

It is also important that Dirac and Majorana neutrinos exhibit quite different electromagnetic properties. For instance, for Dirac neutrinos the millicharge q_{if}, the magnetic moment μ_{if}, the electric dipole moment ε_{if} and the anapole moment a_{if} in general could be not zero both diagonal and off-diagonal. For Majorana neutrinos all diagonal from factors should be zero.

The hermiticity and the behavior of electromagnetic current under discrete symmetric transformation put very severe bounds on the decomposition of the vertex function.

In the easiest generalization of the Standard Model when we suppose that neutrino is a massive particle the one loop calculation provides the following contribution to the Dirac neutrino magnetic moment:

$$\mu_{jj}^D = \frac{3e_0G_Fm_j}{8\sqrt{2}\pi^2} \approx 3.2\cdot 10^{-19}\mu_B\left(\frac{m_i}{1\ eV}\right), \tag{11.33}$$

if we scale the mass of mass state of neutrino on the level of $1\ eV$, which is the limit that we know from KATRIN experiment. We obtained the really tiny value that is many orders of magnitude lower than the present experimental bounds from GEMMA (2012):

$$\mu_\nu^{eff} < 2.9\times 10^{-11}\mu_B, \tag{11.34}$$

Borexino (2017):

$$\mu_\nu^{eff} < 2.8 \times 10^{-11}\mu_B \tag{11.35}$$

and astrophysical experiments by Raffelt (1988, 2020) and Arcoa Dias (2015):

$$\mu_\nu^{eff} < 0.1 \times 10^{-11}\mu_B \tag{11.36}$$

The millicharge of neutrino is also constrained by the reactor neutrinos scattering experiment (2014):

$$q_{\nu_e} < \sim 10^{-12}e_0, \tag{11.37}$$

astrophysical experiments:

$$q_{\nu_e} < \sim 10^{-19}e_0, \tag{11.38}$$

neutrality of the hydrogen atom:

$$q_{\nu_e} < \sim 10^{-21}e_0 \tag{11.39}$$

It is also very important not to forget that there is a charge radius of neutrino that is not zero even in the Standard Model and it is the most accessible neutrino electromagnetic property for experimental observation.

Future prospects

There is a set of experiments by the XENON collaboration; studying solar neutrino fluxes they put the limit on neutrino magnetic moment on the level of $10^{-11}\mu_B$. About three years ago they claimed that they even observed the magnetic moment in the region of

$$\mu_\nu \in (1.4, 2.9) \times 10^{-11}\mu_B \tag{11.40}$$

But with the next series of experiments XENONnT they presented the following limit:

$$\mu_\nu < \sim 10^{-12}\mu_B \tag{11.41}$$

There are very severe bounds on neutrino magnetic moment from astrophysics $\sim 10^{-12}\mu_B$. There are also many attempts to improve the present laboratory bounds. I would like to advertise the project that is now preparing in the National Center for Physics and Mathematics in Sarov. This experiment is based on our prediction published in Rep.Phys.D in 2019 together with our Italian colleagues. We’ve proposed a new type of an experiment, the goal of this experiment is to measure the scattering of tritium antineutrinos on atomic liquid helium target. In this scheme of experiment it is possible to lower the sensitivity to the value of neutrino magnetic moment. We predict that the achievable scale of neutrino magnetic moment will be

$$\mu_\nu < 7 \times 10^{-13}\mu_B \tag{11.42}$$

Lecture 12. Neutrino electromagnetic properties in experiments

Introduction

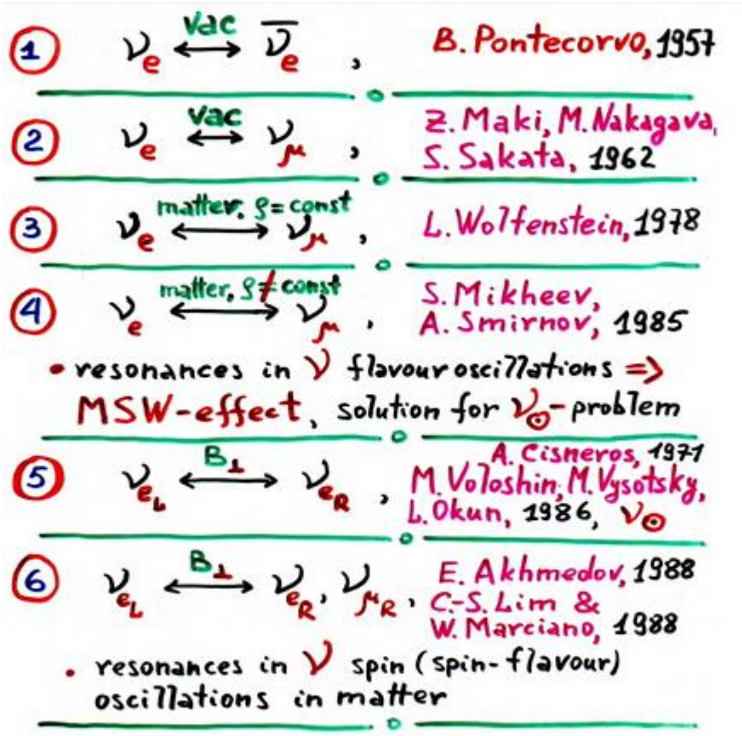
This lecture is dedicated to the studying of neutrino electromagnetic properties in laboratory experiments and constraints on neutrino magnetic and electric dipole moments μ_ν and d_ν, neutrino millicharge q_ν and neutrino charge radius $\langle r_\nu^2 \rangle$.

Firstly, I would like to advertise my paper published together with Carlo Giunti from University of Torino in Italy. The title of this paper is “Neutrino electromagnetic interactions: A window to new physics”. It was published in Reviews of Modern Physics in 2015. This is really the most complete summary of neutrino electromagnetic properties. I also suggest you to look through an appendix to this paper where a lot of very interesting technical details are discussed.

Historical introduction

The famous Bruno Pontecorvo’s paper “Mesonium and anti-mesonium” was published in 1957 in the Soviet physics journal JETP. This was the starting point of neutrino mixing and oscillation effect. In this paper he wrote: “It was assumed above that there exists a conservation law for the neutrino charge, according to which a neutrino cannot change into an antineutrino in any approximation. This law has not yet been established; evidently it has been merely shown that the neutrino and antineutrino are not identical particles. If the two-component neutrino theory should turn out to be incorrect ... and if the conservation law of neutrino charge would not apply, then in principle neutrino - antineutrino transitions could take place in vacuum”. I would like to recall that at that time in 1957 there was no experimental confirmation of the existence of different types of neutrinos and now we know there are electron, muon and tau neutrinos. The second type flavor neutrino was discovered in 1962. So if people spoke about different types of neutrinos in 1957 they meant neutrino and antineutrino. Also as we know if two-component theory is not applicable it means that one should use the four-component neutrino theory and that neutrino is a massive particle.

That was the first theoretical discovery of the effect of neutrino mixing. The summary is that if the mass of neutrino is not zero then neutrino of different types (neutrino and antineutrino) can mix. That prediction of Bruno Pontecorvo was made also in 1957 in his next paper “Inverse β-processes and nonconservation of leptonic charge”. He wrote: “Neutrinos in vacuum can transform themselves into antineutrino and vice versa. This means that neutrino and antineutrino are particle mixtures... So, for example, a beam of neutral leptons from a reactor which at first consists mainly of antineutrinos will change its composition and at a certain distance R from the reactor will be composed of neutrino and antineutrino in equal quantities”. This last sentence can be considered as the prediction of the effect of neutrino oscillation. So the neutrino mixing and oscillations effects were predicted by Bruno Pontecorvo 67 years ago.



Pic. 12.1. The main steps in neutrino oscillations study.

I'd like to recall shortly the main steps in neutrino oscillations study. Firstly, Bruno Pontecorvo proposed the mixing and oscillations in 1957. Then in 1962 after the discovery of the second type of neutrino which is the muon neutrino the group of Japanese scientists Sakata, Maki and Nakagava using the idea of Bruno Pontecorvo proposed the effect of oscillations in vacuum between two types of neutrino, electron and muon. However, they just stated that neutrinos can be considered as mix states but they didn't go further and discuss the evolution of the type of neutrino during the neutrino flux propagation from the source to the detector. In 1969 Gribov and Pontecorvo for the first time derived the probability of neutrino oscillations, the probability to observe the muon neutrino in the initial flux totally composed of electron neutrinos. Ten years later in 1978 American scientist Leon Wolfenstein considered the real situation of neutrino propagation inside quite dense matter in the Sun. He studied the effect of neutrino scattering on particles of matter. Then in 1985 Mikheev and Smirnov who by the way graduated from the Faculty of Physics of the Moscow State University they considered the effect of neutrino oscillations in matter and have found that for particular value of matter density there could be a huge increase of the probability of mixing and oscillations. This effect was called the effect of resonance amplification of neutrino flavor oscillations in matter or the MSW-effect (Mikheev-Smirnov-Wolfenstein). Then in 1971 A. Cisneros for the first time considered the effect of mixing and oscillations between different neutrino spin states, electron left and electron right neutrinos. As we know neutrino has nontrivial magnetic moment, this is just a consequence of the effect of non-zero neutrino mass. So these neutrinos interact with the magnetic field and in presence of transversal magnetic field in respect to neutrino propagation there are mixing and oscillations between left and right neutrinos. This effect can be resonantly increased by the presence of matter. This phenomenon has been theoretically studied independently by E. Akhmedov and C.-S.Lim and W.Marciano in 1988.

Neutrino oscillations in vacuum

Pontecorvo proposed that the flavor states $\nu^{(f)} = \begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$ are linear combinations of the mass states or physical states $\nu^{(p)} = \begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$. Japanese colleagues parameterized this mixing by the only parameter of mixing angle θ_ν:

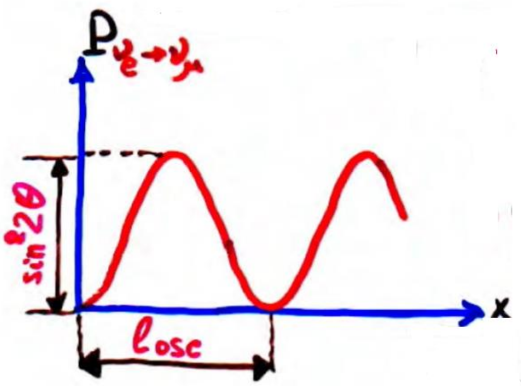
$$\nu_e = \nu_1 \cos\theta_\nu + \nu_2 \sin\theta_\nu, \quad (12.1)$$

$$\nu_\mu = -\nu_1 \cos\theta_\nu + \nu_2 \sin\theta_\nu \quad (12.2)$$

They also introduced the mixing matrix which is now called PMNS matrix (Pontecorvo-Maki-Nacagawa-Sakata):

$$U = \begin{pmatrix}\cos\theta_\nu & \sin\theta_\nu\\ -\sin\theta_\nu & \cos\theta_\nu\end{pmatrix}, \quad (12.3)$$

$$\nu^{(f)} = U\nu^{(p)} \quad (12.4)$$



Pic.12.2. The probability of neutrino oscillations.

I have already mentioned that Gribov and Pontecorvo derived the probability of neutrino oscillations which is equal to the probability of finding muon neutrino ν_μ in an initial flux of electron neutrinos ν_e that travel a distance x from the source to the detector:

$$\mathrm{P}_{\nu_e\nu_\mu}(x) = \left|\langle\nu_\mu|\nu_e\rangle_t\right|^2 = \sin^2 2\theta_\nu \sin^2\left(\frac{\pi x}{L_\nu}\right), \quad (12.5)$$

$$L_\nu = \frac{4\pi E}{|m_1^2 - m_2^2|}, E \cong |\vec{p}|, \quad (12.6)$$

where the length of oscillations L_ν is determined by the energy of neutrinos and the mass square difference. The amplitude of oscillations is equal to $\sin^2 2\theta_\nu$.

Neutrino oscillations in matter

In 1985 Mikheev and Smirnov using the results of Wolfenstein's calculation of the effect of neutrino scattering on matter predicted that for a particular value of matter density there will be a huge increase of the oscillations amplitude

$$\sin^2 2\theta_{eff} = \frac{\Delta^2 \sin^2 2\theta}{(\Delta \cos 2\theta - A)^2 + \Delta^2 \sin^2 2\theta} \tag{12.6}$$

In particular the resonance will take place in case when

$$\Delta \cos 2\theta = A \tag{12.7}$$

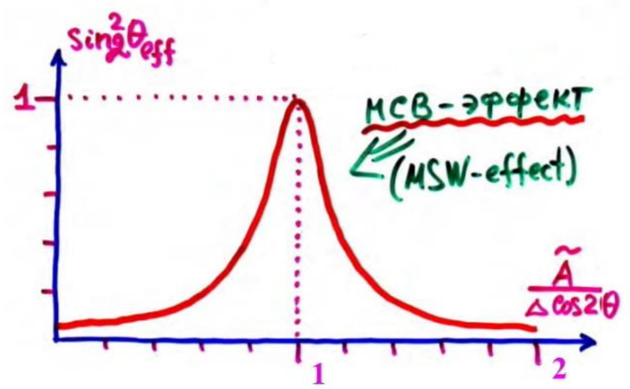
Where A is proportional to the effective number density n_{eff}:

$$A = \sqrt{2} G_F n_{eff}, n_{eff} = n_e \tag{12.8}$$

and

$$\Delta = \frac{\delta m_\nu^2}{2E} \tag{12.9}$$

This effect was called the MSW-effect (Mikheev-Smirnov-Wolfenstein).



Pic.12.3. The MSW-effect.

Neutrino spin and spin-flavor oscillations in transversal magnetic field

I've already mentioned that if neutrino has non-zero mass then inevitably it has nontrivial magnetic moment. The additional mixing appears due to the following effective Lagrangian that couples together right-handed and left-handed neutrinos with different spin states:

$$L \sim \bar{\nu} \sigma_{\lambda\rho} F^{\lambda\rho} \nu' = \bar{\nu}_L \sigma_{\lambda\rho} F^{\lambda\rho} \nu'_R + \bar{\nu}_R \sigma_{\lambda\rho} F^{\lambda\rho} \nu'_L, \tag{12.10}$$

where $F^{\lambda\rho}$ – the tensor of electromagnetic field. As a result it is realized that the effects of spin and spin-flavor oscillations appear in nature. These types of oscillations can also be increased by the resonance effect. It was calculated for the first time by E. Akhmedov and C.-S.Lim and W.Marciano in 1988. The probability of neutrino spin oscillations in presence of transversal magnetic field:

$$P(\nu_L \leftrightarrow \nu_R) = \sin^2 2\theta_{eff} \sin^2\left(\frac{\pi x}{L_{eff}}\right), \tag{12.11}$$

$$\sin^2 2\theta_{eff} = \frac{(2\mu B_\perp)^2}{(2\mu B_\perp)^2 + \Omega^2}, \tag{12.12}$$

$$\Omega = \frac{\Delta m_\nu^2}{2E_\nu} A(\theta_{vac}) - \sqrt{2} G_F n_{eff} \tag{12.13}$$

We can see that the resonance in neutrino spin oscillations appears when $\Omega^2 \to 0$.

The probability of spin-flavor oscillations in presence of transversal magnetic field can be also written as

$$P_{\nu_L \nu_R} = \sin^2\beta \sin^2 \Omega z, \tag{12.14}$$

$$\sin^2\beta = \frac{\left(2\mu_{e\mu}B\right)^2}{\left(2\mu_{e\mu}B\right)^2 + \left(\frac{\Delta_{LR}}{4E}\right)^2}, \tag{12.15}$$

$$\Delta_{LR} = \frac{\Delta m^2}{2}(\cos 2\theta + 1) - 2EV_{\nu_e} + 2E\dot{\phi}, \tag{12.16}$$

where $\theta -$ vacuum mixing angle, $V_{\nu_e} -$ matter potential and the third term accounts for the possible variation of constant magnetic field along the trajectory of neutrino, it's so-called twisting magnetic field:

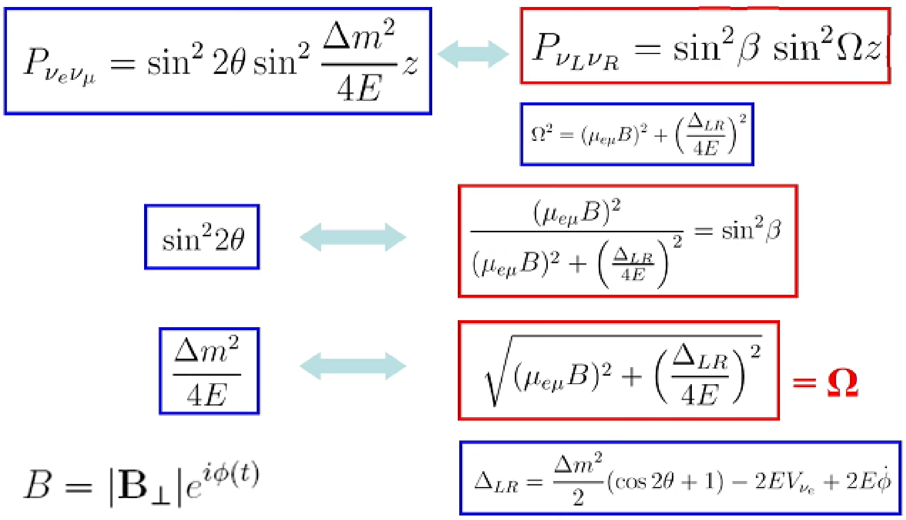
$$B = \left|\vec{B}_\perp\right| e^{i\phi(t)} \tag{12.17}$$

So there is a resonance amplification of neutrino spin-flavor oscillations in matter. This effect appears when

$$\Delta_{LR} \to 0 \Rightarrow \tag{12.18}$$

$$\sin^2\beta \to 1 \tag{12.19}$$

There is a correspondence between the flavor oscillations and spin or spin-flavor oscillations.



Pic.12.4. The correspondence between the flavor and spin-flavor oscillations.

Neutrino magnetic moment

The most easily accepted both from theoretical and experimental points of view among the neutrino electromagnetic properties are dipole magnetic and electric moments. However the most acceptable for the future experimental observation is the neutrino charge radius $\langle r_\nu^2 \rangle$. The studies of neutrino-electron scattering provide the most sensitive method for experimental investigation of nontrivial neutrino magnetic moment. I would like to recall once again that up to now there are no any experimental confirmations from laboratory experiments as well as from astrophysical observations in favor of nontrivial neutrino electromagnetic properties. It means that all experimental data is in agreement with the hypothesis that there are no any neutrino electromagnetic properties. So all electromagnetic characteristics such as dipole magnetic and electric moments and charge radius of neutrino are zero. But we know for sure that if neutrino is a massive particle which is confirmed experimentally then inevitably it has at least non-zero magnetic moment. Also even in case when the mass of neutrino is zero the charge radius of neutrino can be not zero.

The cross-section of neutrino on electrons is composed of two terms:

$$\frac{d\sigma}{dT}(\nu + e \to \nu + e) = \left(\frac{d\sigma}{dT}\right)_{SM} + \left(\frac{d\sigma}{dT}\right)_{\mu_\nu} \tag{12.20}$$

The first one is the Standard Model contribution:

$$\left(\frac{d\sigma}{dT}\right)_{SM} = \frac{G_F^2 m_e}{2\pi}\left[(g_V + g_A)^2 + (g_V - g_A)^2\left(1 - \frac{T}{E_\nu}\right)^2 + (g_A^2 - g_V^2)\frac{m_e T}{E_\nu^2}\right], \tag{12.21}$$

$$g_V = \begin{cases} 2\sin^2\theta_W + \dfrac{1}{2} & for\ \nu_e \\ 2\sin^2\theta_W - \dfrac{1}{2} & for\ \nu_\mu, \nu_\tau \end{cases}, \tag{12.22}$$

$$g_A = \begin{cases} \dfrac{1}{2} & for\ \nu_e \\ -\dfrac{1}{2} & for\ \nu_\mu, \nu_\tau \end{cases} \tag{12.23}$$

The second term can appear if neutrino magnetic moment is not zero:

$$\left(\frac{d\sigma}{dT}\right)_{\mu_\nu} = \frac{\pi\alpha_{em}^2}{m_e^2}\left[\frac{1 - \frac{T}{E_\nu}}{T}\right]\mu_\nu^2, \tag{12.24}$$

it's proportional to the effective neutrino magnetic moment given by the following expression which accounts for the effect of mixing:

$$\mu_\nu^2(\nu_l, L, E_\nu) = \sum_j \left|\sum_i U_{li} e^{-iE_i L}\mu_{ji}\right|^2 \tag{12.25}$$

The most correct way is to introduce neutrino electromagnetic characteristics for the mass states. But we know that in experiment the flavor neutrinos are detected. So we should derive effective electromagnetic characteristics for the flavor states. The effective magnetic moment μ_ν in (12.25) is given as a function of fundamental magnetic moments μ_{ji} which are introduced for the mass states of neutrino.

It is possible also to incorporate the effect of charge radius by simple shifting the vector g_V by adding the additional term that is proportional to the charge radius:

$$g_V \to g_V + \frac{2}{3} M_W^2 \langle r^2 \rangle \sin^2\theta_W \tag{12.26}$$

If neutrino has nontrivial electric dipole moment or electric or magnetic transition moments, these quantities also contribute to the scattering cross-section:

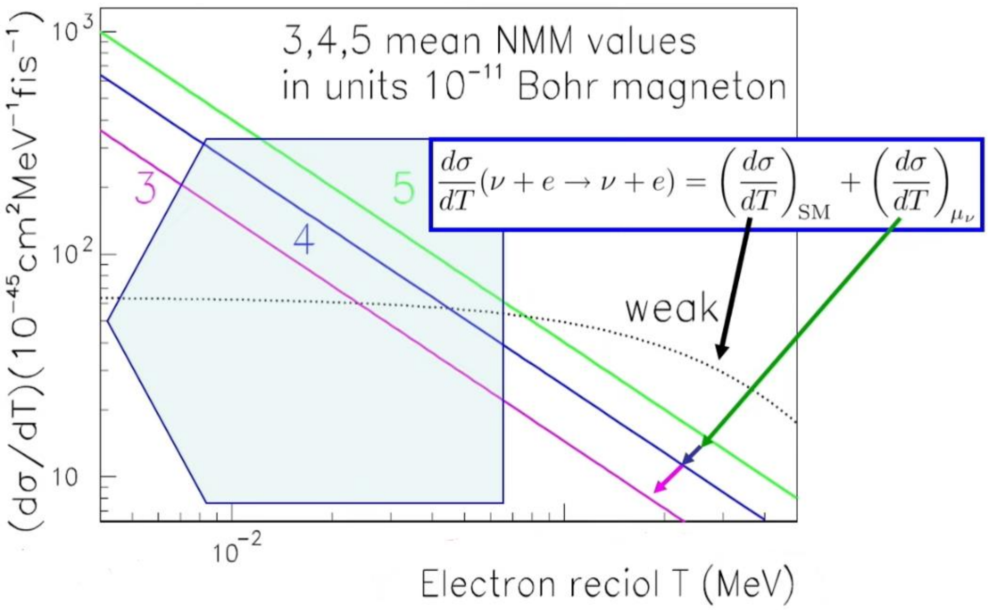
$$\mu_{\nu_l}^2 \cong \sum_j \left| \sum_k U_{lk}^* \left(\mu_{jk} - i\epsilon_{jk}\right) \right|^2 \tag{12.27}$$

Once again, neutrino effective magnetic moment is measured in neutrino-electron scattering. Firstly, we consider an electron neutrino ν_e as a supposition of mass states of neutrino at some distance L from the source, and then we sum up magnetic moment contributions to ν-e scattering amplitude (of each of mass components) induced by their magnetic moments:

$$A_j \sim \sum_i U_{ei} e^{-iE_i L} \mu_{ji} \tag{12.28}$$

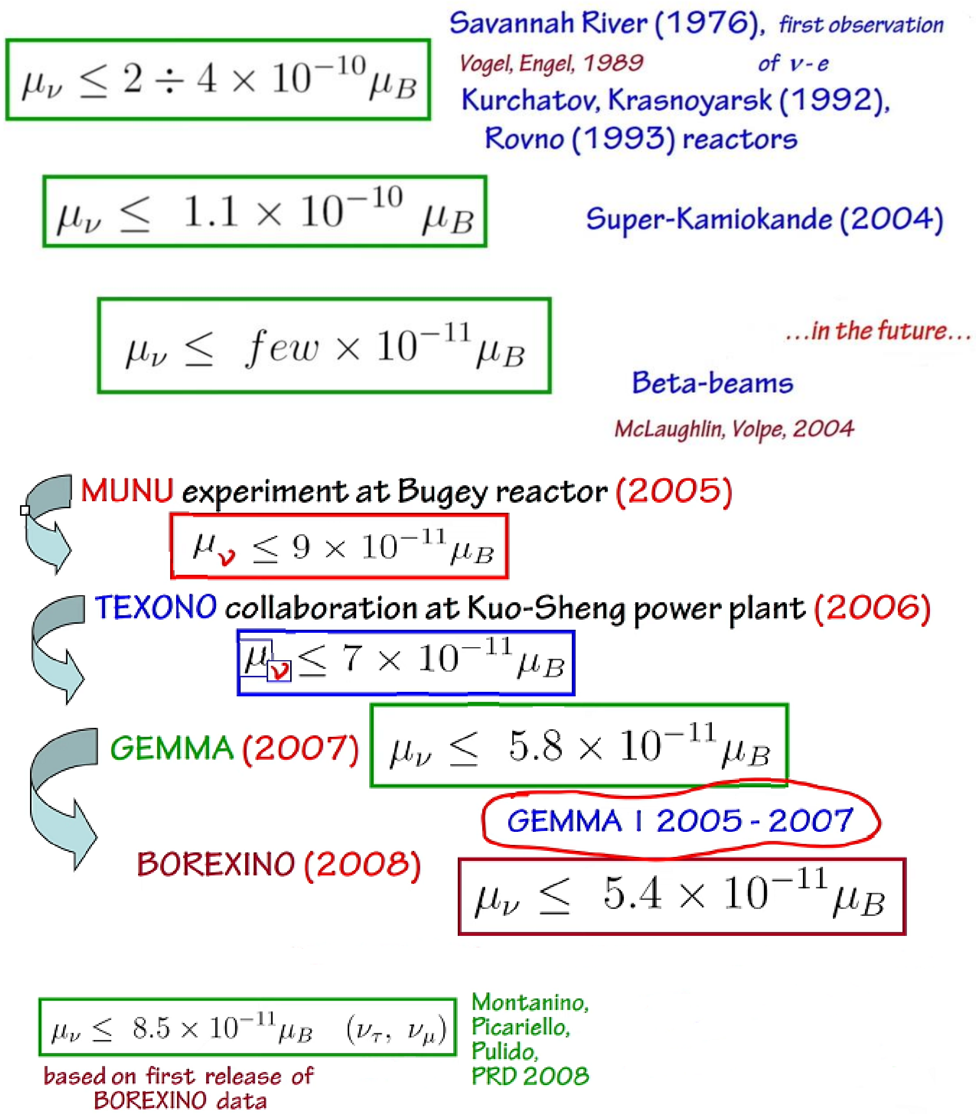
These amplitudes combine incoherently in the total cross-section:

$$\sigma \sim \mu_e^2 = \sum_j \left| \sum_i U_{ei} e^{-iE_i L} \mu_{ji} \right|^2 \tag{12.29}$$



Pic. 12.5. The cross-section dependence on the energy of electron.

Picture 12.5 illustrates the behavior of the two contributions to the neutrino-electron scattering. We see that magnetic moment contribution dominates at low electron recoil energies T when $\left(\frac{d\sigma}{dT}\right)_{\mu_\nu} > \left(\frac{d\sigma}{dT}\right)_{SM}$ and $\frac{T}{m_e} < \frac{\pi^2\alpha_{em}}{G_F^2 m_e^4}\mu_\nu^2$. And also the lower the smallest measurable electron recoil energy is, the smaller values of μ_ν^2 can be probed in scattering experiments.



Pic. 12.6. First and future neutrino-electron scattering experiments.

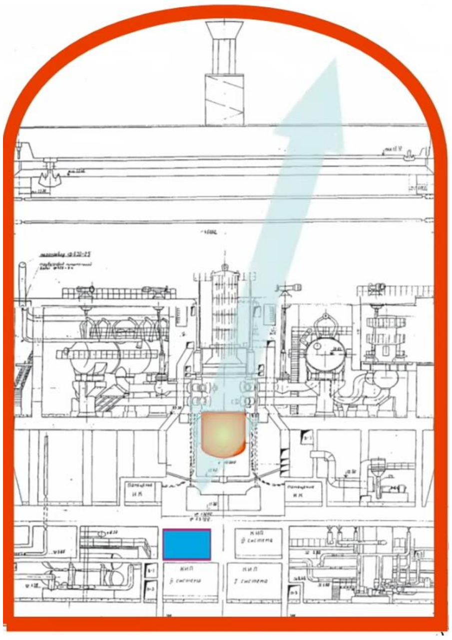
All the experiments from the very beginning up to now only provide an upper limit for the effective magnetic moment of neutrino. The most severe bound has been obtained in investigation of the solar fluxes by GEMMA experiment (Germanium Experiment for Measurement of Magnetic Moment of Antineutrino). This experiment has been running for more than 10 years under the control of JINR (Dubna) and ITEP (Moscow) at Kalinin

Nuclear Power Plant. Since 2012 they provide the world best experimental limit on the magnetic moment of neutrino:

$$\mu_\nu < 2.9 \times 10^{-11} \mu_B \qquad (12.30)$$

The next addition to this GEMMA experiment is called GEMMA-2 or νGeN experiment will be sensitive to the magnetic moment of neutrino on the value of

$$\mu_\nu \sim (5 - 9) \times 10^{-12} \mu_B \qquad (12.31)$$

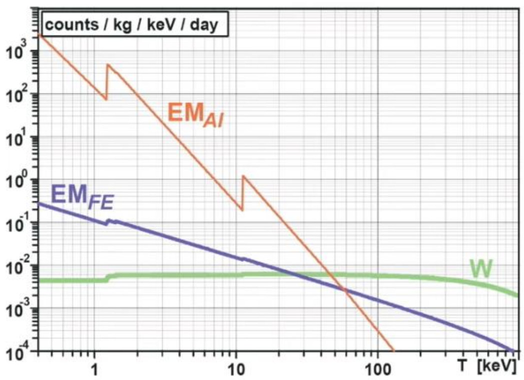


Pic.12.7. Reactor unit of the Kalinin Nuclear Power Plant.

The sensitivity to the effect of magnetic moment is determined by the following expression:

$$\mu_\nu \sim \frac{1}{\sqrt{N_\nu}} \left(\frac{B}{mt}\right)^{\frac{1}{4}}, \qquad (12.32)$$

$N_\nu -$ number of signal events expected, $B -$ background level in the ROI, $m -$ target (detector) mass, $t -$ measurement time.



Pic.12.8. The result of TEXONO experiment.

In 2010 just before the Neutrino Conference of that year the TEXONO experiment group claimed that they provided a new and more correct theoretical estimation of the contribution of magnetic moment effect to the scattering. They stated that neutrino-electron cross-section should be increased by atomic ionization effect. It means that electrons are not free particles but they are bound in atom. So TEXONO group have calculated the scattering of neutrinos on bound electrons. They obtained new much better limits on the neutrino effective magnetic moment: $\mu_\nu < 1.3 \times 10^{-11}\mu_B$ on TEXONO experimental data and $\mu_\nu < 5.0 \times 10^{-12}\mu_B$ on GEMMA data. However, together with my colleagues K.Kouzakov and M.Voloshin in the series of papers we provide very detailed calculation of atomic ionization effect. We have shown that for particular scheme of experiment that is realized in TEXONO experiment there is no important effect of atomic ionization on cross-section. Free electron approximation is quite enough to derive the contribution of neutrino magnetic moment to the cross-section. So we just confirmed that GEMMA results were correct.

GEMMA-2 experiment was moved from one reactor to another and also the detector was installed on the lifting platform. It increased the sensitivity of the experiment to neutrino magnetic moment.

I would like to attract your attention to a very important fact that what measured in the scattering experiments is an effective magnetic moment that depends not only on fundamental magnetic moments of mass states but also on a distance between the source and the detector. Indeed, an observable neutrino magnetic moment is an effective parameter that depends on neutrino flavor composition at the detector. Therefore implications of neutrino magnetic moment limits from different experiments are different.

The effects of mixing and oscillations were studied in details in our paper published in Phys.Rep.D by me together with K.Kouzakov. We considered the electromagnetic vertex function of neutrinos and derived the closest expressions for all form factors and electromagnetic characteristics such as neutrino millcharge, anapole moment, magnetic and electric dipole moments. I only would like to make a summary of these calculations.

In particular, it's possible to introduce the so-called generalized neutrino charge. In mass basis:

$$\tilde{Q}_{jk} = \frac{2\sqrt{2}\pi\alpha}{G_F}\left[\frac{(e_\nu)_{jk}}{q^2} + \frac{1}{6}\langle r_\nu^2\rangle_{jk}\right] \quad (12.33)$$

In flavor basis:

$$\tilde{Q}_{l'l} = \sum_{j,k=1}^{3} U_{l'j}U_{lk}^*\tilde{Q}_{jk} = \frac{2\sqrt{2}\pi\alpha}{G_F}\left[\frac{(e_\nu)_{l'l}}{q^2} + \frac{1}{6}\langle r_\nu^2\rangle_{l'l}\right] \quad (12.34)$$

There are contributions from an electric charge:

$$(e_\nu)_{l'l} = \sum_{j,k=1}^{3} U_{l'j}U_{lk}^*(e_\nu)_{jk} \quad (12.35)$$

and a charge radius:

$$\langle r_\nu^2\rangle_{l'l} = \sum_{j,k=1}^{3} U_{l'j} U_{lk}^* \langle r_\nu^2\rangle_{jk} \tag{12.36}$$

Another result of our studies is the confirmation the statement that depending on the base line corresponding to different experiments different combinations of fundamental magnetic moments can be constrained. For instance, in the short baseline experiments such as GEMMA experiment the effect of neutrino flavor change is insignificant. What is really measured is the value for an effective magnetic moment of neutrino that is determined by the fundamental magnetic moments introduced for the mass states:

$$(\mu_\nu)_{l'l} = \sum_{j,k=1}^{3} U_{lk}^* U_{l'j} (\mu_\nu)_{jk} \tag{12.37}$$

For the long baseline experiments such as Borexino experiment there is another combination of fundamental magnetic moments:

$$|\mu_\nu(L, E_\nu)|^2 = \sum_{j,k=1}^{3} |U_{lk}|^2 \left|(\mu_\nu)_{jk}\right|^2 \tag{12.38}$$

As one of the examples of possibility to constrain magnetic moment of neutrino with the long baseline experiments I would like to mention the Borexino experiment that was running up to recent time in Gran Sasso National Laboratory in Italy. The main goal was to detect the solar neutrinos. In this experiment the effective magnetic moment was constrained on the value of $2.8 \times 10^{-11} \mu_B$ which is very close to the result of GEMMA experiment.

Method	Experiment	Limit	CL	Reference
	Krasnoyarsk	$\mu_{\nu_e} < 2.4 \times 10^{-10}\,\mu_B$	90%	Vidyakin *et al.* (1992)
	Rovno	$\mu_{\nu_e} < 1.9 \times 10^{-10}\,\mu_B$	95%	Derbin *et al.* (1993)
Reactor $\bar{\nu}_e$-e^-	MUNU	$\mu_{\nu_e} < 0.9 \times 10^{-10}\,\mu_B$	90%	Daraktchieva *et al.* (2005)
	TEXONO	$\mu_{\nu_e} < 7.4 \times 10^{-11}\,\mu_B$	90%	Wong *et al.* (2007)
	GEMMA	$\mu_{\nu_e} < 2.9 \times 10^{-11}\,\mu_B$	90%	Beda *et al.* (2012)
Accelerator ν_e-e^-	LAMPF	$\mu_{\nu_e} < 10.8 \times 10^{-10}\,\mu_B$	90%	Allen *et al.* (1993)
Accelerator $(\nu_\mu, \bar{\nu}_\mu)$-e^-	BNL-E734	$\mu_{\nu_\mu} < 8.5 \times 10^{-10}\,\mu_B$	90%	Ahrens *et al.* (1990)
	LAMPF	$\mu_{\nu_\mu} < 7.4 \times 10^{-10}\,\mu_B$	90%	Allen *et al.* (1993)
	LSND	$\mu_{\nu_\mu} < 6.8 \times 10^{-10}\,\mu_B$	90%	Auerbach *et al.* (2001)
Accelerator $(\nu_\tau, \bar{\nu}_\tau)$-$e^-$	DONUT	$\mu_{\nu_\tau} < 3.9 \times 10^{-7}\,\mu_B$	90%	Schwienhorst *et al.* (2001)
Solar ν_e-e^-	Super-Kamiokande	$\mu_S(E_\nu \gtrsim 5\,\text{MeV}) < 1.1 \times 10^{-10}\,\mu_B$	90%	Liu *et al.* (2004)
	Borexino	$\mu_S(E_\nu \lesssim 1\,\text{MeV}) < 5.4 \times 10^{-11}\,\mu_B$	90%	Arpesella *et al.* (2008)

Pic.12.9. The list of the most important constraints on the effective magnetic moment.

Neutrino electric charge

In the Standard Model it's proved that neutrino is an electrically neutral particle. This result is attributed to two important statements: gauge invariance and the demand of anomaly cancellation constraints. In the Standard Model if we consider the case of massless neutrino (so this is the model without right-handed neutrinos) the triangle anomalies cancellation constraints put a certain relations among particle hypercharges that is enough to fix all hypercharges Y so that they, and consequently the charge Q, are quantized. Once the charge is quantized there is no room for neutrino millicharge. The fact that $Q = 0$ has been proven also by direct calculations in SM within different gauges and methods. But strict requirements for Q quantization may disappear in extensions of the Standard Model when right-handed neutrinos are included: in the absence of Y quantization electric charge Q gets dequantized. So only experimental results put a limit on neutrino electric charge.

About 10 years ago I have added the third unobserved term in the cross-section due to possible nontrivial neutrino electric charge:

$$\left(\frac{d\sigma}{dT}\right)_{\nu-e} = \left(\frac{d\sigma}{dT}\right)_{SM} + \left(\frac{d\sigma}{dT}\right)_{\mu_\nu} + \left(\frac{d\sigma}{dT}\right)_{q_\nu}, \tag{12.39}$$

$$\left(\frac{d\sigma}{dT}\right)_{q_\nu} \approx 2\pi\alpha\frac{1}{m_e T^2}q_\nu^2 \tag{12.40}$$

In order to be in agreement with the experimental data from the GEMMA experiment it is possible to get an upper bound for neutrino millicharge:

$$|q_\nu| < 1.5 \times 10^{-12} e_0 \tag{12.41}$$

With the new edition of GEMMA experiment or νGeN that started in 1992 and running now we expect

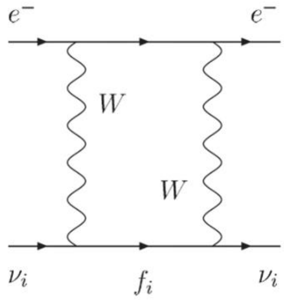
$$|q_\nu| < 1.1 \times 10^{-13} e_0 \tag{12.42}$$

The present limit from GEMMA data (12.41) is also included to the table of neutrino properties by the Particle Data Group collaboration.

Neutrino charge radius

Neutrino charge radius is the most accessible for experimental investigations among the other electromagnetic properties. The charge radius is often claimed to be related with the anapole moment. In literature we can come across the claim that in the Standard Model the anapole moment is proportional to the charge radius. But in our paper we discussed that these two electromagnetic properties are not connected in such a simple way.

I would like to mention a series of papers and theoretical investigations by Bernabeu and collaborators who tried to solve the problem of defining the charge radius as a finite, not divergent and physical value. Because if you make theoretical calculations for the charge radius in case of non-zero neutrino mass the result will be not finite and even gauge dependent. Bernabeu and collaborators include additional diagrams that also contribute to the scattering of neutrinos on charge leptons. They also introduced the electro-week charge radius that indeed can be determined as a physical value that is not dependent on gauge fix parameter and finite.



Pic.12.10. Box diagrams of neutrino scattering.

Referring to our investigation we have shown that neutrino charge radius in neutrino-electron scattering experiments cannot be considered as a shift to the vector coupling constant

$$g_V \to g_V + \frac{2}{3} M_W^2 \langle r^2 \rangle \sin^2 \theta_W , \tag{12.43}$$

there are other contributions from the flavor-transition charge radius. Recently together with our Chinese and Italian colleagues we have investigated the possibility of manifestation of the not diagonal charge radius of neutrinos. We implemented this possibility in the study and for the first time derived the constraints on the non-diagonal charge radius:

$$\left(\langle r^2_{\nu_{e,\mu}}\rangle , \langle r^2_{\nu_{e,\tau}}\rangle, \langle r^2_{\nu_{\mu,\tau}}\rangle\right) < (28,30,35) \times 10^{-32} cm^2 \tag{12.44}$$

These constraints were highlighted by the editors of Physical Review D as the most important result published in this journal during the year.

Method	Experiment	Limit (cm^2)	C.L.	Reference
Reactor $\bar{\nu}_e$-e^-	Krasnoyarsk	$\lvert\langle r^2_{\nu_e}\rangle\rvert < 7.3 \times 10^{-32}$	90%	Vidyakin *et al.* (1992)
	TEXONO	$-4.2 \times 10^{-32} < \langle r^2_{\nu_e}\rangle < 6.6 \times 10^{-32}$	90%	Deniz *et al.* (2010)[a]
Accelerator ν_e-e^-	LAMPF	$-7.12 \times 10^{-32} < \langle r^2_{\nu_e}\rangle < 10.88 \times 10^{-32}$	90%	Allen *et al.* (1993)[a]
	LSND	$-5.94 \times 10^{-32} < \langle r^2_{\nu_e}\rangle < 8.28 \times 10^{-32}$	90%	Auerbach *et al.* (2001)[a]
Accelerator ν_μ-e^-	BNL-E734	$-4.22 \times 10^{-32} < \langle r^2_{\nu_\mu}\rangle < 0.48 \times 10^{-32}$	90%	Ahrens *et al.* (1990)[a]
	CHARM-II	$\lvert\langle r^2_{\nu_\mu}\rangle\rvert < 1.2 \times 10^{-32}$	90%	Vilain *et al.* (1995)[a]

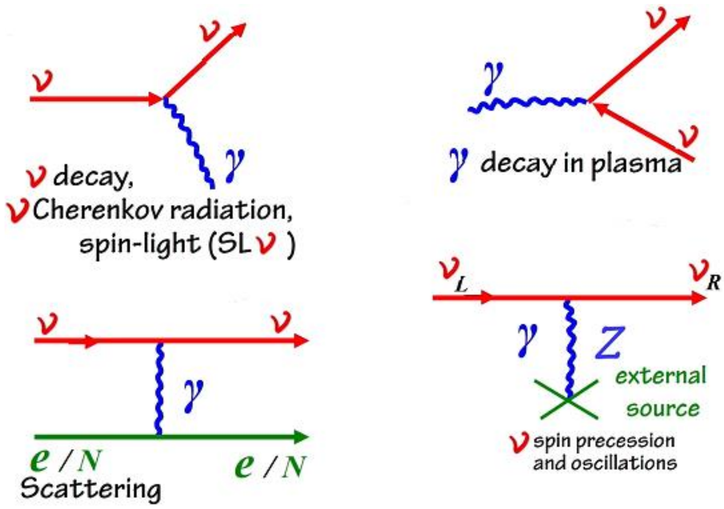
Pic.12.11. The list of constraints on neutrino charge radius.

Lecture 13. Electromagnetic neutrinos: new effects in magnetic fields and matter

Introduction

We are going to discuss the influence of external electromagnetic fields and matter on neutrinos. One of the interesting effects is that in some cases the influence of background matter on neutrino mimics the effect of neutrino interaction with the external electromagnetic fields. Once again I would like to advertise my paper "Neutrino electromagnetic interactions: A window to new physics" published together with Carlo Giunti in Reviews of Modern Physics in 2015. I also suggest you to look through an appendix where a lot of very interesting technical details are discussed.

Neutrino electromagnetic interactions

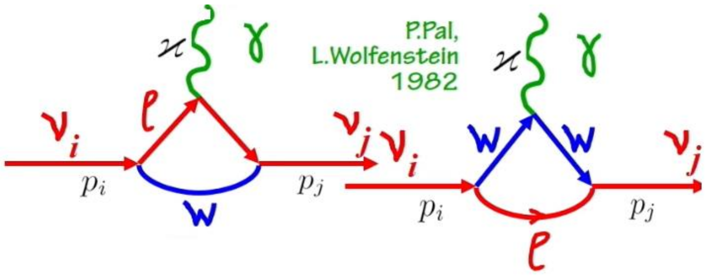


Pic. 13.1. Neutrino electromagnetic interactions.

I would like to summarize the most important effects that arise due to nontrivial neutrino electromagnetic properties. First of all, if neutrino is a massive particle even in the easiest generalization of the Standard Model it should inevitably have nontrivial electromagnetic properties, at least nonzero magnetic moment. So if we consider neutrino a massive particle it can decay to a lighter state of neutrino with emission of a photon. The same diagram in background environment will describe another phenomenon of particular different nature; it can be called neutrino-Cherenkov radiation. Neutrino moving in background matter or in external electromagnetic fields can emit with the same mechanism as electron does. There is another very important phenomenon also described by the same diagram which was for the first time proposed in our papers, so-called spin-light of neutrino (SLν). Neutrino can emit this spin-light when it's moving in dense external matter and again if it has non-trivial electromagnetic properties. Another effect is photon decay to neutrino antineutrino pair in plasma. This is a very important phenomenon for application in astrophysics because its studies provide the best astrophysical bound on neutrino magnetic moment. Once again I would like to recall that up to now neither from terrestrial laboratory experiments nor from astrophysical and cosmology observations there are no any indications of neutrino electromagnetic properties. All the experiments only provide an upper limit on neutrino electromagnetic properties. Another important manifestation of electromagnetic properties of neutrinos is the electromagnetic scattering of neutrinos on electrons or nuclei due to exchange of photons. There is also phenomenon of neutrino spin precession and corresponding oscillations in external electromagnetic fields. For the first time it was proposed and studied theoretically in our papers about 20 years ago. In this process neutrino changes its spin state, so neutrino spin oscillations take place or spin-flavor oscillations if the flavor is also changing. Normally it was believed that the spin or spin-flavor oscillations can proceed only in the presence of transversal magnetic field. But it was proposed and studied theoretically in our papers that there is another possibility of generalization of the spin or spin-flavor oscillations due to interaction with moving matter.

Neutrino magnetic moment

Once again there are diagonal and transition magnetic moments of neutrinos. Let's consider the easiest generalization of the Standard Model that we should apply when we study neutrinos with nonzero mass.



Pic. 13.2. Loop-diagrams.

The typical diagrams that were for the first time considered by Paul and Wolfenstein in 1982 are shown on pic.13.2. The calculation of these loop-diagrams inevitably provides the effect of nonzero magnetic or even electric dipole moments:

$$\left.\begin{matrix}\mu_{ij}\\ \epsilon_{ij}\end{matrix}\right\} = \frac{eG_F m_i}{8\sqrt{2}\pi^2}\left(1 \pm \frac{m_j}{m_i}\right)\sum_{l=e,\mu,\tau} f(r_l)U_{lj}U_{li}^*, \qquad (13.1)$$

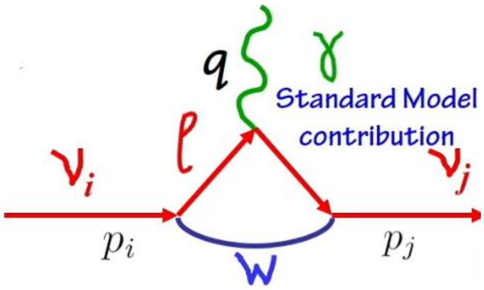
$$r_l = \left(\frac{m_l}{m_W}\right)^2 \qquad (13.2)$$

If we consider $i = j$ there are just normal diagonal magnetic moments in the mass basis. If we consider $i \neq j$ we get that there are transition or non-diagonal magnetic moments in the mass basis. Due to well-known Glashow-Iliopoulos-Maiani cancellation condition and the unitarity of the mixing matrix U we can see from the following formula

$$f(r_l) \approx \frac{3}{2}\left(1 - \frac{1}{2}r_l\right)^2 \qquad (13.3)$$

that only the second term here will contribute. However the transition moments are strongly suppressed in respect to the diagonal moments. For example, if we consider the effect of neutrino decay that we discussed earlier we see that it is quite rare phenomenon because this coupling with the photon happens due to transition magnetic moment.

Neutrino radiative decay



Pic. 13.3. Neutrino radiative decay diagram.

Let's consider a particular process of neutrino radiative decay

$$\nu_i \to \nu_j + \gamma \tag{13.4}$$

This process was calculated for the first time in 1977 by Sergey Petkov. The effective interaction Lagrangian:

$$\mathcal{L}_{int} = \frac{1}{2}\bar{\psi}_i \sigma_{\mu\nu}\left(\mu_{if} + i\gamma_5 d_{if}\right)\psi_j F^{\mu\nu} + h.c. \tag{13.5}$$

The effective contribution to the vertex function:

$$\Lambda_\mu^{if}(q) = -i\sigma_{\mu\nu} q^\nu \left(\mu_{if} + i\gamma_5 d_{if}\right) \tag{13.6}$$

Quite simple calculation gives the rate of this process that depends on the effective magnetic moment of neutrino squared:

$$\Gamma_{\nu_i \to \nu_j + \gamma} = \frac{\mu_{eff}^2}{8\pi}\left(\frac{m_i^2 - m_j^2}{m_i^2}\right)^3, \tag{13.7}$$

$$\mu_{eff}^2 = \left|\mu_{ij}\right|^2 + \left|\epsilon_{ij}\right|^2, \tag{13.8}$$

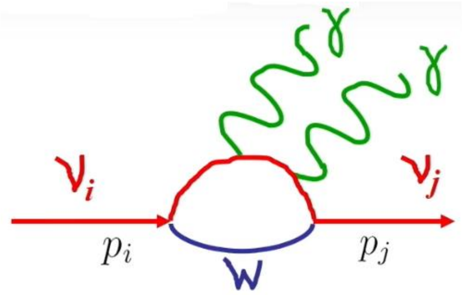
μ_{ij} – transition magnetic moment, ϵ_{ij} – transition dipole electric moment. If we put numbers just to understand the scale of this phenomenon it is possible to calculate the inversed rate that gives us the life time of initial neutrino in respect to this process:

$$\Gamma_{\nu_i \to \nu_j + \gamma} \approx 5\left(\frac{\mu_{eff}}{\mu_B}\right)^2 \left(\frac{m_i^2 - m_j^2}{m_i^2}\right)^3 \left(\frac{m_i}{1\ eV}\right)^3 s^{-1} \Rightarrow \tag{13.9}$$

$$\tau_{\nu_i \to \nu_j + \gamma}^{rf} \approx 0.19 \left(\frac{m_i^2}{m_i^2 - m_f^2}\right)^3 \left(\frac{eV}{m_i}\right)^3 \left(\frac{\mu_B}{\mu_{fi}^{eff}}\right)^2 s \tag{13.10}$$

We see that lifetime is very huge it means that this process is very slow. This process can be applied to different situations that can be found in astrophysics. This radiative decay can be constrained and we can have bounds on the effective magnetic moments of neutrino from different observations:

1) reactor $\bar{\nu}_e$ and solar ν_e fluxes
2) SN 1987 ν burst
3) spectral distortion of CMBR



Pic. 13.4. Neutrino radiative two-photon decay diagram.

There is another similar process of neutrino radiative two-photon decay:

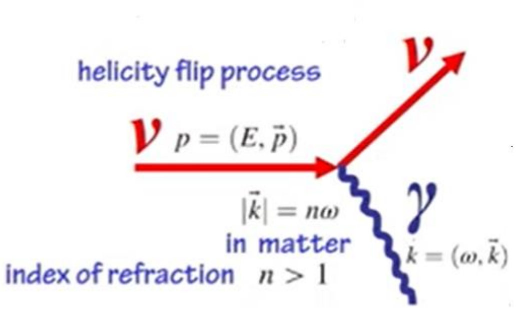
$$\nu_i \to \nu_j + \gamma + \gamma \tag{13.11}$$

An initial neutrino is converted to a lighter final state with the mission of two photons. This diagram is suppressed by an additional coupling with a second photon:

$$\Gamma_{\nu_i \to \nu_j + \gamma + \gamma} \sim \frac{\alpha_{QED}}{4\pi} \Gamma_{\nu_i \to \nu_j + \gamma} \tag{13.12}$$

α_{QED} – fine structure constant that is much less than one: $\alpha_{QED} \ll 1$. There is no GIM cancellation in this process.

Neutrino Cherenkov radiation



Pic. 13.5. Neutrino Cherenkov radiation diagram.

Neutrino Cherenkov radiation appears in presence of external matter. This matter is characterized by the refraction index n that is bigger than one. The initial neutrino is converted to the final with emission of photon:

$$\nu_L(p) \to \nu_R(p') + \gamma(k) \tag{13.13}$$

The Cherenkov rate is given by the following expression:

$$\Gamma = \frac{1}{2(2\pi)^2 E} \int \frac{d^3p'}{2E'} \frac{d^3k}{2\omega} |M|^2 \delta^4(p - p' - k) \qquad (13.14)$$

After straightforward calculation we obtain the next quite not complicated expression that depends on the refraction index of matter and the energy:

$$\Gamma = \frac{\mu^2}{4\pi E^2 v} \int_{\omega_{min}}^{\omega_{max}} \left\{ \left[\frac{(n^2-1)^2}{n^2} E^2 + (n^2-1) m_\nu^2 \right] \omega^2 - \frac{(n^2-1)^2}{n^2} E\omega^3 - \frac{(n^2-1)^3}{4n^2} \omega^4 \right\} d\omega \qquad (13.15)$$

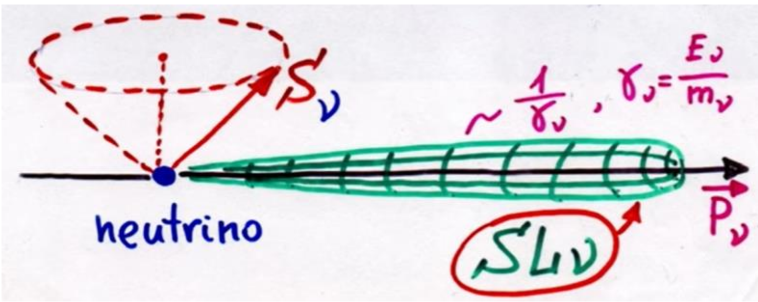
It is possible to make quite simple estimation applying this result for solar neutrinos in terrestrial experiments. If we use the value of the effective magnetic moment on the order of the present terrestrial constraints ($\mu_\nu \sim 3 \times 10^{-11} \mu_B$) for the huge water detractors of the volume of $1km^3$ it will be several photons per day.

There are a vast variety of other radiative decays of the Cherenkov type that can proceed in some cases. I would like to point your attention on neutrino Cherenkov radiation in magnetic field. In this case there is no need to go beyond the Standard Model. The magnetic field induces an additional effective coupling between neutrinos and photons and also modifies the dispersion relation for a photon. For the first time it was discussed in papers published by our colleagues from the Department of Theoretical Physics Galtsov and Nikitina in 1972. Then it was also investigated in some details by Raffelt and Ioannisian in 1997. There is another possibility that in medium neutrino can acquire an induced millicharge due to weak interactions. So neutrino can act like an electron moving in matter. Once again there is no need to go beyond the Standard Model so this process can proceed even in case when the mass of neutrino is zero. So it means that neutrino can be described quite well within the Standard Model.

Neutrino Cherenkov radiation can appear when in addition to external media also electromagnetic fields are present. There is a superposition of the effect of electromagnetic fields on neutrinos and the effect of neutrino interaction with background matter.

Spin-light of neutrino

I would like to mention a quite new mechanism of electromagnetic radiation that is also possible when neutrino has non-trivial electromagnetic properties, magnetic moment in particular, that allows it to be coupled to photons. This effect was proposed for the first time in our papers published together with Andrey Lobanov about 20 years ago. We called this new mechanism of electromagnetic radiation of neutrino the spin-light of neutrino in matter SLν. At first we developed the quasi-classical approach for description of the neutrino spin evolution in an external environment, including the possibility of presence of external electromagnetic fields and external background matter. Later in series of our papers we developed the quantum description of this phenomenon.



Pic. 13.6. Neutrino spin precession in background environment.

We considered the possibility of the spin-light not only in presence of background matter but also in external gravitational fields.

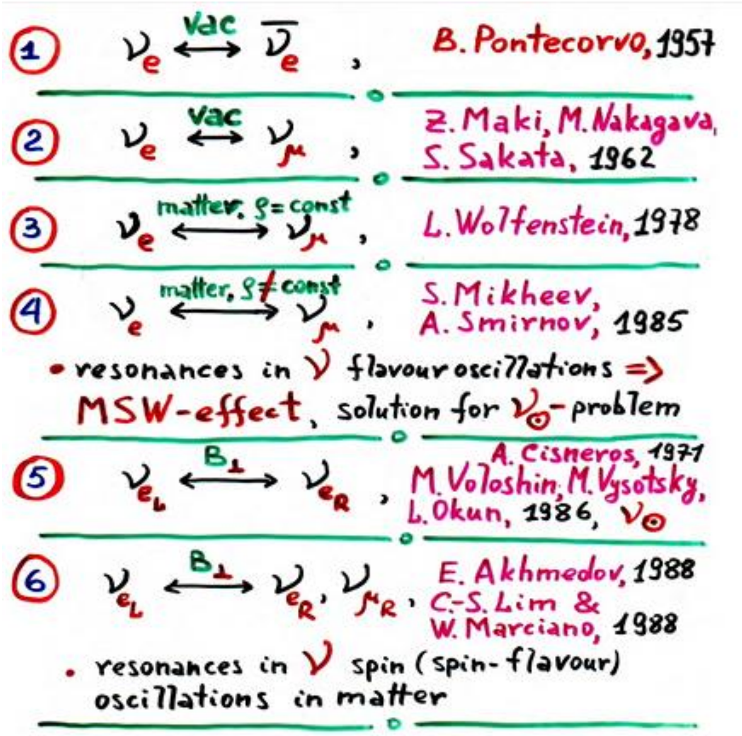
There is an analogy with the classical electrodynamics where an object with zero charge but non-zero magnetic moment can emit light. This is the magnetic dipole radiation and its power is proportional to the second derivative of magnetic moment of the system squared:

$$I = \frac{2}{3}\ddot{\vec{m}}^2 \tag{13.16}$$

The quasi-classical description of this mechanism presented in our paper "Spin Light of Neutrino" published in 2003. I would like to mention that in this process neutrino change its spin state, so the active left-handed neutrino converts to the sterile right-handed neutrino with the emission of light. The quantum theory of spin light of neutrino described in one of our latest papers "Spin light of neutrino in astrophysical environments" published in the Journal of Cosmology and Astroparticle Physics in 2017. We examined different possibilities for the spin light of neutrino manifestation in different astrophysical environments and we have found the cases where it is possible to expect that such type of radiation could be observed in terrestrial experiments.

I'd like to shortly recall the main steps in neutrino oscillations study. Firstly, Bruno Pontecorvo proposed the mixing and oscillations in 1957. Then in 1962 after the discovery of the second type of neutrino which is the muon neutrino the group of Japanese scientists Sakata, Maki and Nakagava using the idea of Bruno Pontecorvo proposed the effect of oscillations in vacuum between two types of neutrino, electron and muon. In 1978 American scientist Leon Wolfenstein calculated the effect of neutrino scattering on background matter. Then in 1985 Mikheev and Smirnov using Wolfenstein's result discovered the effect of resonance amplification of neutrino flavor oscillations due to neutrino scattering on background matter or the MSW-effect (Mikheev-Smirnov-Wolfenstein). For understanding of the spin light of neutrino it's important that very similar to the mixing and oscillations between different flavor neutrinos there are mixing and oscillations between neutrinos with different spin orientations (if we consider neutrino is a massive particle with nonzero magnetic moment). Cisneros in 1971 for the first time predicted the mixing and oscillations between different neutrino spin states, electron left and electron right neutrinos, due to neutrino's interaction with transversal magnetic field in respect to neutrino propagation.

M.Voloshin, M. Vysotsky and L. Okun considered this phenomenon trying to solve solar problem in 1866. In 1988 independently E. Akhmedov and C.-S.Lim, W.Marciano proposed the phenomenon very similar to MSW-effect: the resonance amplification of spin-flavor oscillations in case when neutrino propagates through matter.



Pic. 13.7. The main steps in neutrino oscillations study.

Neutrino spin and spin-flavor oscillations

We have derived in a very short way the probability of neutrino spin oscillations in the transversal magnetic field:

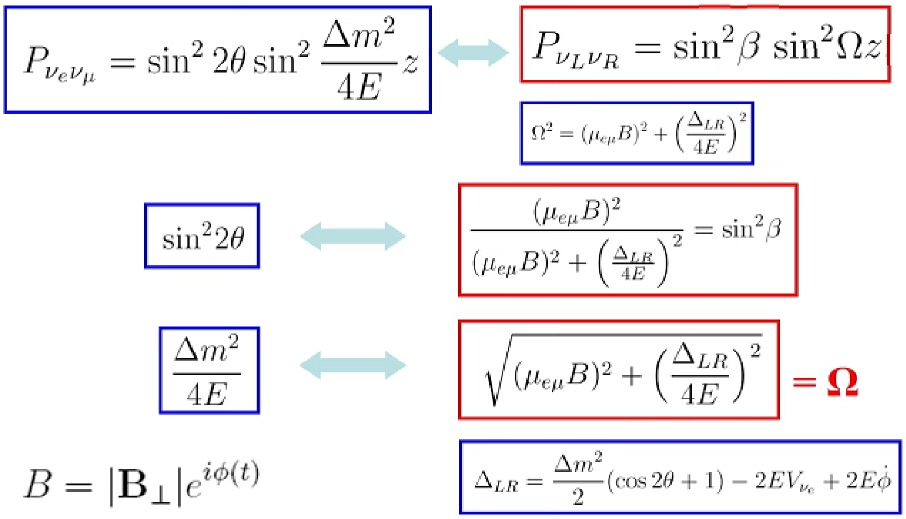
$$P(\nu_L \leftrightarrow \nu_R) = \sin^2 2\theta_{eff} \sin^2\left(\frac{\pi x}{L_{eff}}\right), \quad (13.17)$$

$$\sin^2 2\theta_{eff} = \frac{(2\mu B_\perp)^2}{(2\mu B_\perp)^2 + \Omega^2}, \quad (13.18)$$

$$\Omega = \frac{\Delta m_\nu^2}{2E_\nu} A(\theta_{vac}) - \sqrt{2} G_F n_{eff}, \quad (13.19)$$

$$L_{eff} = \frac{2\pi}{\sqrt{\Omega^2 + (2\mu B_\perp)^2}} \quad (13.20)$$

When $\Omega^2 \to 0$ the oscillations amplitude (13.18) gets its maximal value equal to one.



Pic.13.8. The correspondence between the flavor and spin-flavor oscillations.

There is a correspondence between the flavor oscillations and spin or spin-flavor oscillations. In this process active left-handed neutrino is converted to sterile right-handed neutrino. If you observe neutrinos from the Sun you will get a visible result. Spin and spin-flavor oscillations in magnetic field are important for astrophysical applications.

I am speaking about spin and spin-flavor oscillations because discussing this particular matter on one of the conferences I was asked: "Whether a photon is emitted during the process of neutrino spin or spin flavor oscillations?" My reply to this question was quite not certain, I said that probably the photon should be emitted but the process of this emission is very-very rare and not very important. After that we together with Andrey Lobanov have come to the conclusion that indeed when neutrino propagates in external environment the photon should be emitted as well as when neutrino propagates in external electromagnetic field or in magnetic field in particular.

I would like to consider neutrino spin and spin-flavor oscillations not in constant transversal in respect to neutrino propagation magnetic field but in case of arbitrary magnetic field. In our papers we considered neutrino mass states ($\alpha, \alpha' = 1{,}2$) with two helicities $s = \pm 1$ that propagate in constant magnetic field with transversal and longitudinal components in respect to neutrino propagation $\vec{B} = \vec{B}_\perp + \vec{B}_{||}$. The effective Hamiltonian:

$$H_{\mu_\nu} = -\frac{1}{2}\mu_{\alpha\alpha'}\bar{\nu}_{\alpha'}\sigma_{\mu\nu}\nu_\alpha F^{\mu\nu} + h.c. = -\frac{1}{2}\mu_{\alpha\alpha'}\bar{\nu}_{\alpha'}\sum \vec{B}\,\nu_\alpha, + h.c. \qquad (13.21)$$

The evolution equation for two mass states with two different helicities:

$$i\frac{d}{dt}\begin{pmatrix} \nu_{1,s=1} \\ \nu_{1,s=-1} \\ \nu_{2,s=1} \\ \nu_{2,s=-1} \end{pmatrix} =$$

$$=\frac{1}{2}\begin{pmatrix} E_1+\mu_{11}\frac{B_{\|}}{\gamma_1} & \mu_{11}B_{\perp} & \mu_{12}\frac{B_{\|}}{\gamma_{12}} & \mu_{12}B_{\perp} \\ \mu_{11}B_{\perp} & E_1-\mu_{11}\frac{B_{\|}}{\gamma_1} & \mu_{12}B_{\perp} & -\mu_{12}\frac{B_{\|}}{\gamma_{12}} \\ -\mu_{12}\frac{B_{\|}}{\gamma_{12}} & -\mu_{12}B_{\perp} & E_1+\mu_{22}\frac{B_{\|}}{\gamma_2} & \mu_{22}B_{\perp} \\ -\mu_{12}B_{\perp} & -\mu_{12}\frac{B_{\|}}{\gamma_{12}} & \mu_{22}B_{\perp} & E_1-\mu_{22}\frac{B_{\|}}{\gamma_2} \end{pmatrix}\begin{pmatrix} \nu_{1,s=1} \\ \nu_{1,s=-1} \\ \nu_{2,s=1} \\ \nu_{2,s=-1} \end{pmatrix} \quad (13.22)$$

First of all, we can see that the mixing between two different helicity states happens due to neutrino interaction with transversal magnetic field – $\mu_{11}B_{\perp}$. Secondly, coupling with the longitudinal magnetic field shift the energy of neutrino – $\mu_{11}\frac{B_{\|}}{\gamma_1}$. Thirdly, the mixing between different mass states happens due to interaction of transition magnetic moment with longitudinal magnetic field – $\mu_{12}\frac{B_{\|}}{\gamma_{12}}$.

If we make a transition to the flavor basis:

$$\nu_f=\begin{pmatrix} \nu_e^R \\ \nu_e^L \\ \nu_\mu^R \\ \nu_\mu^L \end{pmatrix}, \quad (13.23)$$

$$\nu_e^{R,L}=\nu_{1,s=\pm1}\cos\theta+\nu_{2,s=\pm1}\sin\theta, \quad (13.24)$$

$$\nu_\mu^{R,L}=-\nu_{1,s=\pm1}\sin\theta+\nu_{2,s=\pm1}\cos\theta, \quad (13.25)$$

we obtain the following magnetic moment interaction Hamiltonian for flavor neutrino:

$$\tilde{H}_B^f=\begin{pmatrix} \left(\frac{\mu}{\gamma}\right)_{ee}B_{\|} & \mu_{ee}B_{\perp} & \left(\frac{\mu}{\gamma}\right)_{e\mu}B_{\|} & \mu_{e\mu}B_{\perp} \\ \mu_{ee}B_{\perp} & -\left(\frac{\mu}{\gamma}\right)_{ee}B_{\|} & \mu_{e\mu}B_{\perp} & -\left(\frac{\mu}{\gamma}\right)_{e\mu}B_{\|} \\ \left(\frac{\mu}{\gamma}\right)_{e\mu}B_{\|} & \mu_{e\mu}B_{\perp} & \left(\frac{\mu}{\gamma}\right)_{\mu\mu}B_{\|} & \mu_{\mu\mu}B_{\perp} \\ \mu_{e\mu}B_{\perp} & -\left(\frac{\mu}{\gamma}\right)_{e\mu}B_{\|} & \mu_{\mu\mu}B_{\perp} & -\left(\frac{\mu}{\gamma}\right)_{\mu\mu}B_{\|} \end{pmatrix}, \quad (13.26)$$

where the effective magnetic moments in flavor basis are

$$\mu_{ee}=\mu_{11}\cos^2\theta+\mu_{22}\sin^2\theta+\mu_{12}\sin 2\theta, \quad (13.27)$$

$$\mu_{\mu\mu}=\mu_{11}\sin^2\theta+\mu_{22}\cos^2\theta-\mu_{12}\sin 2\theta, \quad (13.28)$$

$$\mu_{e\mu}=\mu_{12}\cos 2\theta+\frac{1}{2}(\mu_{22}-\mu_{11})\sin 2\theta, \quad (13.29)$$

$$\left(\frac{\mu}{\gamma}\right)_{ee}=\frac{\mu_{11}}{\gamma_{11}}\cos^2\theta+\frac{\mu_{22}}{\gamma_{22}}\sin^2\theta+\frac{\mu_{12}}{\gamma_{12}}\sin 2\theta, \quad (13.30)$$

$$\left(\frac{\mu}{\gamma}\right)_{\mu\mu} = \frac{\mu_{11}}{\gamma_{11}}\sin^2\theta + \frac{\mu_{22}}{\gamma_{22}}\cos^2\theta - \frac{\mu_{12}}{\gamma_{12}}\sin 2\theta, \qquad (13.31)$$

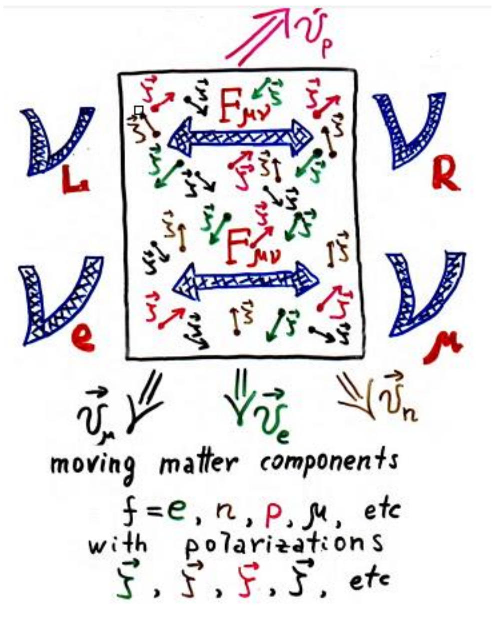
$$\left(\frac{\mu}{\gamma}\right)_{e\mu} = \frac{\mu_{12}}{\gamma_{12}}\cos 2\theta + \frac{1}{2}\left(\frac{\mu_{22}}{\gamma_{22}} - \frac{\mu_{11}}{\gamma_{11}}\right)\sin 2\theta \qquad (13.32)$$

Applying these general expressions for particular case of the oscillations between electron and muon neutrinos with the same spin orientation $\nu_e^L \leftrightarrow \nu_\mu^L$, we can write the effective evolution Hamiltonian:

$$H_B^f = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \tilde{\mu}_{ee}\frac{B_{\parallel}}{\gamma_{ee}} & \frac{\Delta m^2}{4E}\sin 2\theta + \tilde{\mu}_{e\mu}\frac{B_{\parallel}}{\gamma_{e\mu}} \\ \frac{\Delta m^2}{4E}\sin 2\theta + \tilde{\mu}_{e\mu}\frac{B_{\parallel}}{\gamma_{e\mu}} & \frac{\Delta m^2}{4E}\cos 2\theta + \tilde{\mu}_{\mu\mu}\frac{B_{\parallel}}{\gamma_{\mu\mu}} \end{pmatrix} \qquad (13.33)$$

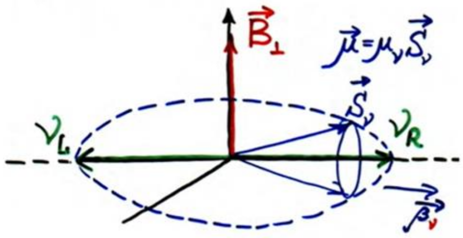
We have found that the longitudinal component of magnetic field modifies the neutrino flavor oscillations, and the transitional component changes the spin orientation and induces the spin and spin-flavor oscillations.

Neutrino spin and spin-flavor oscillations in magnetic field and moving matter



Pic.13.9. Neutrino spin and flavor oscillations in moving matter.

Let's consider neutrino spin and flavor oscillations due to the presence of electromagnetic field and we also account for neutrino scattering on particles of matter. For simplicity we suppose that matter is composed of electrons, neutrons, protons and maybe muons. We characterize each of the matter components by the number density of these particles. We account for the possible motion of matter as a whole. Each of the matter components are also characterized by the polarization.



Pic.13.10. Neutrino spin evolution.

In addition to the well-known from electrodynamics equation for the neutrino spin evolution we get an additional term that accounts for neutrino scattering on particles of the environment or weak interaction of neutrinos with matter:

$$\frac{d\vec{S}_\nu}{dx} = 2\mu_\nu[\vec{S}_\nu \times \vec{B}] + 2\mu_\nu[\vec{S}_\nu \times \vec{G}] \qquad (13.34)$$

Then together with my colleague Maxim Dvornikov we provided quite general investigation of neutrino spin evolution in presence of quite general type of external fields. The Lagrangian that accounts for different types of non-derivative interaction of neutrinos with external fields such as scalar, pseudoscalar, vector, axial-vector, tensor and pseudotensor:

$$-\mathcal{L} = g_s s(x)\bar{\nu}\nu + g_p \pi(x)\bar{\nu}\gamma^5\nu + g_v V^\mu(x)\bar{\nu}\gamma_\mu\nu + g_a A^\mu(x)\bar{\nu}\gamma_\mu\gamma^5\nu$$
$$+\frac{g_t}{2}T^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\nu + \frac{g_t'}{2}\Pi^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\gamma_5\nu, \qquad (13.35)$$

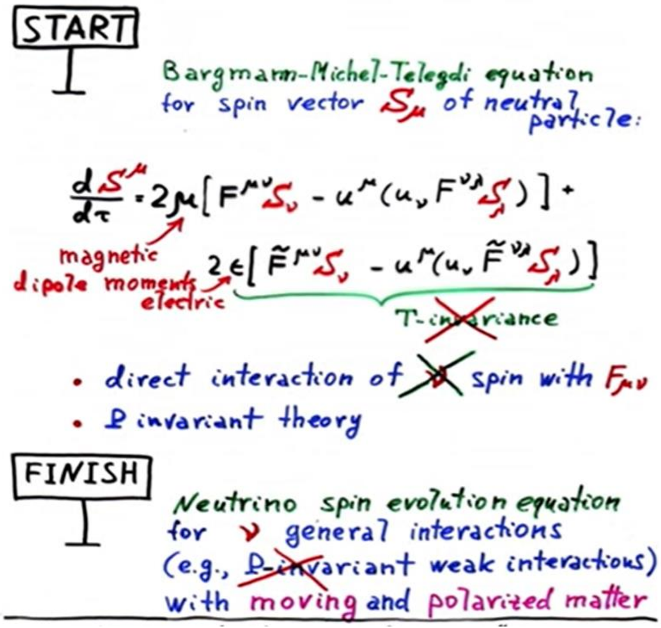
where $s, \pi, V^\mu = (V^0, \vec{V}), A^\mu = (A^0, \vec{A}), T_{\mu\nu} = (\vec{a}, \vec{b}), \Pi_{\mu\nu} = (\vec{c}, \vec{d})$. We have found more general quasi-classical relativistic equation for the neutrino spin evolution:

$$\dot{\vec{\zeta}}_\nu = 2g_a\left\{A^0[\vec{\zeta}_\nu \times \vec{\beta}] - \frac{m_\nu}{E_\nu}[\vec{\zeta}_\nu \times \vec{A}] - \frac{E_\nu}{E_\nu+m_\nu}(\vec{A}\vec{\beta})[\vec{\zeta}_\nu \times \vec{\beta}]\right\}$$
$$+2g_t\left\{[\vec{\zeta}_\nu \times \vec{b}] - \frac{E_\nu}{E_\nu+m_\nu}(\vec{\beta}\vec{b})[\vec{\zeta}_\nu \times \vec{\beta}] + [\vec{\zeta}_\nu \times [\vec{a} \times \vec{\beta}]]\right\} +$$
$$+2ig_t'\left\{[\vec{\zeta}_\nu \times \vec{c}] - \frac{E_\nu}{E_\nu+m_\nu}(\vec{\beta}\vec{c})[\vec{\zeta}_\nu \times \vec{\beta}] - [\vec{\zeta}_\nu \times [\vec{d} \times \vec{\beta}]]\right\}$$

From this it follows that neither scalar s nor pseudoscalar π or vector V contributes to the spin evolution, and only electromagnetic interactions and weak interactions can influence neutrino spin evolution.

Quasi-classical theory of neutrino spin light

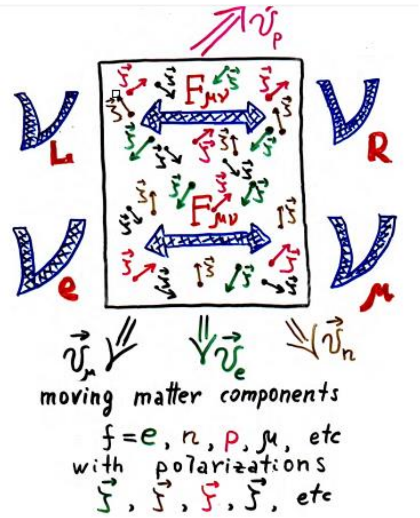
We are now ready to start the discussion on the quasi-classical theory of neutrino spin light that we first developed together with my colleague Andrey Lobanov. But probably we better discuss it in details next time.



Pic.13.11. Neutrino spin evolution in arbitrary electromagnetic field and moving and polarized matter.

Lecture 14. Electromagnetic neutrinos: new effects in magnetic fields and matter 2

Neutrino spin and spin-flavor oscillations in magnetic field and moving matter



Pic.14.1. Neutrino spin and flavor oscillations in moving matter.

We consider two types of flavor neutrinos ν_e, ν_μ with two spin orientations ν_L, ν_R in presence of arbitrary electromagnetic field described by the electromagnetic field tensor $F_{\mu\nu}$. The matter through which neutrinos are propagating is composed of electrons, neutrons, protons and maybe muons. Each of the matter components are characterized by the polarization. There are several papers devoted to neutrino spin and sin-flavor oscillations in magnetic field and moving matter published together with my postgraduate student A. Egorov and my colleague A. Lobanov in 2000-2002. At first we considered this problem using the so called quasi-classical description of neutrino interactions with electromagnetic fields and moving matter. We started with the generalization of the Bargmann-Michel-Telegdi equation for spin vector evolution of a neutral particle:

$$\frac{dS^\mu}{d\tau} = 2\mu\left[F^{\mu\nu}S_\nu - u^\mu\left(u_\nu F^{\nu\lambda}S_\lambda\right)\right] + 2\epsilon\left[\tilde{F}^{\mu\nu}S_\nu - u^\mu\left(u_\nu \tilde{F}^{\nu\lambda}S_\lambda\right)\right], \qquad (14.1)$$

$\mu -$ magnetic moment, $\epsilon -$ dipole electric moment, $F^{\mu\nu} -$ arbitrary electromagnetic field tensor, $u^\mu -$ neutrino's speed vector, $u_\mu = (\gamma, \gamma\vec{\beta})$. We managed to generalize this equation that comes from electrodynamics to the case when neutrino weak interactions with the particles of background are also accounted for. We substitute the electromagnetic field tensor by itself plus the tensor that should account for neutrino interaction with moving and polarized matter:

$$F_{\mu\nu} \to F_{\mu\nu} + G_{\mu\nu} \tag{14.2}$$

The problem was how to get an exact expression for this introduced tensor $G_{\mu\nu}$. For each of the matter components we introduced the four dimensional currents:

$$j_f^\mu = \left(n_f, n_f \vec{v}_f\right), f = n, e, p, \tag{14.3}$$

and polarizations:

$$\lambda_f^\mu = \left(n_f \vec{\zeta}_f \vec{v}_f, n_f \vec{\zeta}_f \sqrt{1 - v_f^2} + \frac{n_f \vec{v}_f \left(\vec{\zeta}_f \vec{v}_f\right)}{1 + \sqrt{1 - v_f^2}} \right), \tag{14.4}$$

$n_f -$ the number density of background particle f, $\vec{v}_f -$ the speed in the reference frame in which the mean momentums of each matter component f is zero, $\vec{\zeta}_f -$ the mean value of polarization vectors of f in above mentioned reference frame. For each of fermions there are only $u_f^\mu, j_f^\mu, \lambda_f^\mu$ to construct $G_{\mu\nu}$. If j_f^μ, λ_f^μ are slowly varying functions in space and time (similar to $F_{\mu\nu}$ in Bargmann-Michel-Telegdi equation) then only four tensors (for each of f) linear in respect to matter charact.:

$$G_1^{\mu\nu} = \epsilon^{\mu\nu\rho\lambda} u_\lambda j_\rho, \tag{14.5}$$

$$G_2^{\mu\nu} = \epsilon^{\mu\nu\rho\lambda} u_\lambda \lambda_\rho, \tag{14.6}$$

$$G_3^{\mu\nu} = u^\mu j^\nu - j^\mu u^\nu, \tag{14.7}$$

$$G_4^{\mu\nu} = u^\mu \lambda^\nu - \lambda^\mu u^\nu \tag{14.8}$$

Thus, in general case of neutrino interaction with different background fermions matter effects are described by antisymmetric tensor:

$$G^{\mu\nu} = \epsilon^{\mu\nu\rho\lambda} g_\rho^{(1)} u_\lambda - \left(g^{(2)\mu} u^\nu - u^\mu g^{(2)\nu}\right), \tag{14.9}$$

where

$$g^{(1)\mu} = \sum_f \rho_f^{(1)} j_f^\mu + \rho_f^{(2)} \lambda_f^\mu, \tag{14.10}$$

$$g^{(2)\mu} = \sum_f \xi_f^{(1)} j_f^\mu + \xi_f^{(2)} \lambda_f^\mu \tag{14.11}$$

We can also express this tensor that accounts for neutrino interactions with the background matter in a way similar to the expression for the electromagnetic field tensor:

$$F_{\mu\nu} = \left(\vec{E}, \vec{B}\right) = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix} \Rightarrow \tag{14.12}$$

$$G_{\mu\nu} = \left(-\vec{P}, \vec{M}\right), \tag{14.13}$$

where

$$\vec{M} = \gamma\left\{ g_0^{(1)} \vec{\beta} - \vec{g}^{(1)} - \left[\vec{\beta} \times \vec{g}^{(2)}\right]\right\}, \tag{14.14}$$

$$\vec{P} = -\gamma\left\{g_0^{(2)}\vec{\beta} - \vec{g}^{(2)} + \left[\vec{\beta}\times\vec{g}^{(1)}\right]\right\} \tag{14.15}$$

The substitution (14.2) means that we actually add to the real electromagnetic field the additional vectors:

$$\vec{B} \to \vec{B} + \vec{M} \tag{14.16}$$

$$\vec{E} \to \vec{E} - \vec{P} \tag{14.17}$$

Finally, in the laboratory frame we get the following generalized Bargmann-Michel-Telegdi equation of the three-dimensional neutrino spin vector evolution accounting for interaction of neutrinos with the magnetic and electric fields:

$$\frac{d\vec{S}}{dt} = \frac{2\mu}{\gamma}\left[\vec{S}\times\left(\vec{B}_0 + \vec{M}_0\right)\right] + \frac{2\epsilon}{\gamma}\left[\vec{S}\times\left(\vec{E}_0 - \vec{P}_0\right)\right], \tag{14.18}$$

where $\vec{B}_0, \vec{M}_0, \vec{E}_0, \vec{P}_0$ are given in the rest frame of neutrino and expressed in terms of quantities determined in laboratory frame:

$$\vec{B}_0 = \gamma\left(\vec{B}_\perp + \frac{1}{\gamma}\vec{B}_{||} + \sqrt{1-\frac{1}{\gamma^2}}\left[\vec{E}_\perp\times\vec{n}\right]\right), \tag{14.19}$$

$$\vec{E}_0 = \gamma\left(\vec{E}_\perp + \frac{1}{\gamma}\vec{E}_{||} - \sqrt{1-\frac{1}{\gamma^2}}\left[\vec{B}_\perp\times\vec{n}\right]\right), \tag{14.20}$$

$$\vec{M}_0 = \gamma\vec{\beta}\left(g_0^{(1)} - \frac{\vec{\beta}\vec{g}^{(1)}}{1+\gamma^{-1}}\right) - \vec{g}^{(1)}, \tag{14.21}$$

$$\vec{P}_0 = -\gamma\vec{\beta}\left(g_0^{(2)} - \frac{\vec{\beta}\vec{g}^{(2)}}{1+\gamma^{-1}}\right) + \vec{g}^{(2)}, \tag{14.22}$$

$\vec{n} = \vec{\beta}/\beta$. Now we should determine $\rho_f^{(1)}$ and $\xi_f^{(1)}$. Using $SM + SU(2) - singlet\ \nu_R$ theory we have the neutrino electroweak interactions Lagrangian:

$$L_{eff} = -f^\mu\left(\bar{\nu}\gamma_\mu\frac{1+\gamma_5}{2}\nu\right), \tag{14.23}$$

where

$$f^\mu = \frac{G_F}{\sqrt{2}}\left((1+\sin^2\theta_W)j_e^\mu - \lambda_e^\mu\right) \tag{14.24}$$

For simplicity we only account for the electron's components. If neutrino dipole electric moment is zero $\epsilon = 0$ and $f_\mu = 2\mu g_\mu^{(1)}$ then

$$\rho_e^{(1)} = \frac{G_F}{2\mu\sqrt{2}}(1+4\sin^2\theta_W), \tag{14.25}$$

$$\rho_e^{(2)} = -\frac{G_F}{2\mu\sqrt{2}} \tag{14.26}$$

If we also account for the prediction of the Standard Model that the neutrino magnetic moment is proportional to its mass as

$$\mu_\nu = \frac{3}{8\sqrt{2}\pi^2} e G_F m_\nu \tag{14.27}$$

we get

$$\rho^{(1)} = \frac{4\pi^2}{3em_\nu}(1 + 4\sin^2\theta_W), \qquad \rho^{(2)} = -\frac{4\pi^2}{3em_\nu} \tag{14.28}$$

Finally, we obtain the following expression for the additional vector to the real magnetic field:

$$\vec{M}_0 = n_e\gamma\vec{\beta}\Big\{\big[\rho^{(1)} + \rho^{(2)}\vec{\zeta}\vec{v}_e\big]\big(1 - \vec{\beta}\vec{v}_e\big) + \\ +\rho^{(2)}\sqrt{1 - v_e^2}\left[\frac{\vec{\zeta}\vec{v}_e\vec{\beta}\vec{v}_e}{1 + \sqrt{1 - v_e^2}} - \vec{\zeta}\vec{\beta}\right] + O(\gamma^{-1})\Big\} \tag{14.29}$$

In slowly moving matter $v_e \ll 1$:

$$\vec{M}_0 = n_e\gamma\vec{\beta}\big(\rho^{(1)} - \rho^{(2)}\vec{\zeta}\vec{\beta}\big), \tag{14.30}$$

where the first term exactly reproduces the Wolfenstein term that was discussed for many times in our previous lectures; the second term accounts for possible polarization effect. In case of relativistic matter $v_e \sim 1$:

$$\vec{M}_0 = n_e\gamma\vec{\beta}\big(\rho^{(1)} + \rho^{(2)}\vec{\zeta}\vec{\beta}\big)\big(1 - \vec{\beta}\vec{v}_e\big) \tag{14.31}$$

We can see that in case when matter is moving in the direction of neutrino propagation the matter effect contribution to neutrino spin evolution equation is suppressed; and when matter is moving in inverse direction in respect to neutrino propagation there is a huge increase of matter effect. We come to the conclusion that indeed the matter motion itself can drastically change the matter effect in neutrino spin and flavor oscillations. Based on these derivations we have form formulated some new effects.

First of all, now we know how to treat neutrino spin oscillations in arbitrary electromagnetic field within Lorentz invariant approach. Before these our studies only the case of constant magnetic field was considered. The formalism that we have derived enables us to consider neutrino mixing and oscillations in various electromagnetic fields (electromagnetic wave, etc.). We have derived the expressions for the probabilities of oscillations in presence of electromagnetic fields and have found that there could be a new type of resonance that is given by the following resonance condition:

$$\frac{V}{2} - \frac{\Delta m^2 A}{4E} - \frac{g\omega}{2}\left(1 - \frac{\beta}{\beta_0}\cos\varphi\right) = 0, \tag{14.32}$$

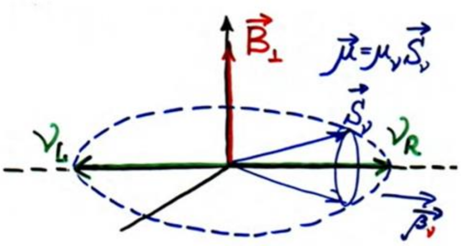
where $V -$ the matter potential, $A = A(\theta_{vac})$ depends on the vacuum mixing angle on the type of transition we are considering, $E -$ the energy of neutrino, $g -$ the polarization of electromagnetic field, $\omega -$ the frequency of electromagnetic field, $\varphi -$ the angle between the neutrino propagation and the propagation of electromagnetic field.

Matter effects

Now we shall fix our attention on the matter effect. In addition to the well-known from electrodynamics equation for the neutrino spin evolution we get an additional term that accounts for neutrino scattering on particles of the environment or weak interaction of neutrinos with matter:

$$\frac{d\vec{S}_\nu}{dx} = 2\mu_\nu[\vec{S}_\nu \times \vec{B}] + 2\mu_\nu[\vec{S}_\nu \times \vec{G}] \tag{14.33}$$

I would like to point that this three-dimensional vector $\vec{G}$ that accounts for weak interactions is inverse proportional to the magnetic moment of neutrino, it means that the second term does not depend on neutrino magnetic moment.



Pic.14.2. Neutrino spin evolution.

Once again let's consider the easiest generalization of the Standard Model $SM + SU(2) - singlet\ \nu_R$ and electron neutrino moving in matter composed of electrons, then the equation for the three-dimensional neutrino spin evolution:

$$\frac{d\vec{S}_\nu}{dt} = \frac{2\mu_\nu}{\gamma_\nu}[\vec{S}_\nu \times (\vec{B}_0 + \vec{M}_0)], \tag{14.34}$$

$$\vec{B}_0 = \gamma_\nu\left(\vec{B}_\perp + \frac{1}{\gamma_\nu}\vec{B}_{||} + \sqrt{1-\frac{1}{\gamma^2}}[\vec{E}_\perp \times \vec{n}]\right), \tag{14.35}$$

$$\vec{M}_0 = \gamma_\nu\rho n_e\left(\vec{\beta}_\nu(1-\vec{\beta}_\nu\vec{v}_e) - \frac{1}{\gamma_\nu}\vec{v}_{e_\perp}\right) \tag{14.36}$$

The first term in (14.36) is longitudinal in respect to neutrino propagation and the second term is transversal. Just from here it follows that neutrino precession $(\nu_L \leftrightarrow \nu_R)$ can be produced by weak interactions of neutrino with the moving matter in case when transversal current of matter is not zero.

There is a study of neutrino spin evolution in presence of general external fields. In our paper we consider the general types of non-derivative interactions of neutrino with external fields such as scalar, pseudoscalar, vector, axial-vector, tensor and pseudotensor given by the following Lagrangian:

$$-\mathcal{L} = g_s s(x)\bar{\nu}\nu + g_p\pi(x)\bar{\nu}\gamma^5\nu + g_v V^\mu(x)\bar{\nu}\gamma_\mu\nu + g_a A^\mu(x)\bar{\nu}\gamma_\mu\gamma^5\nu$$
$$+\frac{g_t}{2}T^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\nu + \frac{g_t'}{2}\Pi^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\gamma_5\nu, \tag{14.37}$$

where $s, \pi, V^{\mu} = (V^0, \vec{V}), A^{\mu} = (A^0, \vec{A}), T_{\mu\nu} = (\vec{a}, \vec{b}), \Pi_{\mu\nu} = (\vec{c}, \vec{d})$. We have found more general quasi-classical relativistic equation for the neutrino spin evolution:

$$\begin{aligned}\dot{\vec{\zeta}}_\nu = 2g_a\left\{A^0[\vec{\zeta}_\nu \times \vec{\beta}] - \tfrac{m_\nu}{E_\nu}[\vec{\zeta}_\nu \times \vec{A}] - \tfrac{E_\nu}{E_\nu+m_\nu}(\vec{A}\vec{\beta})[\vec{\zeta}_\nu \times \vec{\beta}]\right\} \\ +2g_t\left\{[\vec{\zeta}_\nu \times \vec{b}] - \tfrac{E_\nu}{E_\nu+m_\nu}(\vec{\beta}\vec{b})[\vec{\zeta}_\nu \times \vec{\beta}] + [\vec{\zeta}_\nu \times [\vec{a} \times \vec{\beta}]]\right\} + \\ +2ig_t'\left\{[\vec{\zeta}_\nu \times \vec{c}] - \tfrac{E_\nu}{E_\nu+m_\nu}(\vec{\beta}\vec{c})[\vec{\zeta}_\nu \times \vec{\beta}] - [\vec{\zeta}_\nu \times [\vec{d} \times \vec{\beta}]]\right\}\end{aligned}$$

From this it follows that neither scalar s nor pseudoscalar π or vector V contributes to the spin evolution, and only electromagnetic interactions and weak interactions can influence neutrino spin evolution.

In my paper “Neutrino in Electromagnetic Fields and Moving Media” published in 2004 I for the first time considered the possible effect of mixing between neutrinos of different spin orientation due to neutrino weak interactions in case when there is nonzero transversal matter current. I have derived the explicit form for the probability of spin oscillations $\nu_{e_L} \to \nu_{e_R}$ or $\nu_{e_L} \to \nu_{\mu_R}$:

$$P(\nu_i \to \nu_j) = \sin^2 2\theta_{eff} \sin^2\left(\frac{\pi x}{L_{eff}}\right), i \neq j, \tag{14.38}$$

where

$$L_{eff} = \frac{2\pi}{\sqrt{E_{eff}^2 + \Delta_{eff}^2}}, \tag{14.39}$$

$$\sin^2 2\theta_{eff} = \frac{E_{eff}^2}{E_{eff}^2 + \Delta_{eff}^2}, \tag{14.40}$$

$$\Delta_{eff}^2 = \frac{\mu}{\gamma_\nu}\left|\vec{B}_{0_{||}} + \vec{M}_{0_{||}}\right|, \tag{14.41}$$

$$E_{eff} = \mu\left|\vec{B}_{\perp} + \frac{1}{\gamma_\nu}\vec{M}_{0_\perp}\right| \tag{14.42}$$

I also included the possible effects of neutrino spin and spin-flavor oscillations due to the presence of nonzero longitudinal and transversal magnetic field components. From these formulas it follows that even in the absence of magnetic field and magnetic moment there still will be one contribution due to transversal component $\vec{M}_{0_\perp}$ given by the second term in (14.36). It depends on ρ which is inverse proportional to the magnetic moment of neutrino:

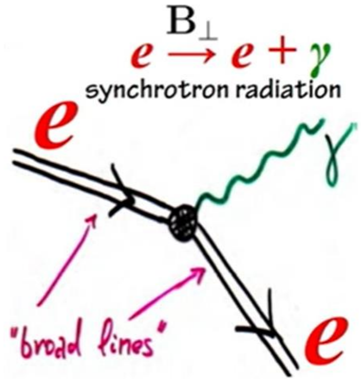
$$\rho = \frac{G_F}{2\mu_\nu\sqrt{2}}(1 + 4\sin^2\theta_W) \tag{14.43}$$

In the conclusion of my paper one can read the following statement: “The possible emergence of neutrino spin oscillations (for example, $\nu_{e_L} \leftrightarrow \nu_{e_R}$) owing to neutrino interaction with matter under the condition that there exists a nonzero transverse current component or matter polarization (that is, $\vec{M}_{0_\perp} \neq 0$) is the most important new effect that follows from the investigation of neutrino spin oscillations in Section 4. So far, it has been assumed that neutrino spin oscillations may arise only in the case where there exists a nonzero transverse magnetic field in the neutrino rest frame." The effect of neutrino helicity conversions and oscillations induced by transversal matter currents has been confirmed in more recent papers. This effect has found some application in problems of neutrino propagation in astrophysical environments.

Spin light of neutrino in matter

Now I would like to discuss the new effect that has been already introduced within the quasi-classical treatment of neutrino propagation in the background media, we called this phenomenon the spin light of neutrino. Neutrino moving in dense matter can emit light due to nontrivial electromagnetic properties, in particular nonzero neutrino magnetic moment.

At first we developed the quasi-classical treatment to this phenomenon but obviously it is a quantum effect. To develop the quantum theory of spin light we generalized the well-known *method of exact solutions* from quantum electrodynamics to the case when neutrino is moving in the background matter. This method implies that at first stage exact solutions for the particle wave functions are found whether in presence of matter or in presence of electromagnetic fields.

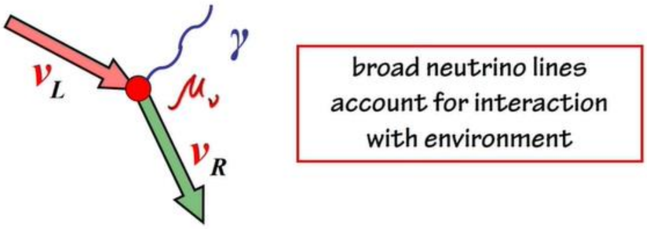


Pic.14.3. Synchrotron radiation.

This method was developed mostly for application to the theory of synchrotron radiation. According to the method of exact solutions for the initial and final electron states we use the exact solutions for the Dirac equation in electromagnetic field. The Dirac equation for electron wave function accounting for interaction with magnetic field:

$$\left\{\gamma^\mu \left(i\partial_\mu - eA_\mu^{ext}(x)\right) - m_e\right\}\psi_F(x) = 0 \qquad (14.44)$$

We generalized this method to neutrino moving in external electromagnetic fields and then for neutrino interaction with the dense matter. It enables us to find exact expressions for neutrino wave functions and neutrino energies in different environments. We can introduce the neutrino quantum states in presence of magnetic field and dense matter. Although this method was developed for the needs of the neutrino spin light effect in matter and within this method we also have found another very interesting phenomenon that is the neutrino energy quantization in rotating matter.



Pic.14.4. The neutrino spin light.

In the series of our papers we have found an exact solution for the neutrino wave function:

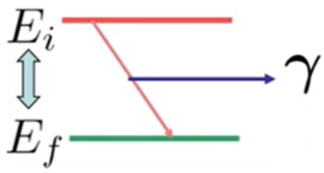
$$\left\{i\gamma_\mu\partial^\mu - \frac{1}{2}\gamma_\mu(1+\gamma_5)f^\mu - m\right\}\psi_F(x) = 0, \qquad (14.45)$$

where f^μ – the matter term that contains matter current j^μ and matter polarization λ^μ:

$$f^\mu = \frac{G_F}{\sqrt{2}}\left((1+4\sin^2\theta_W)j^\mu - \lambda^\mu\right) \qquad (14.46)$$

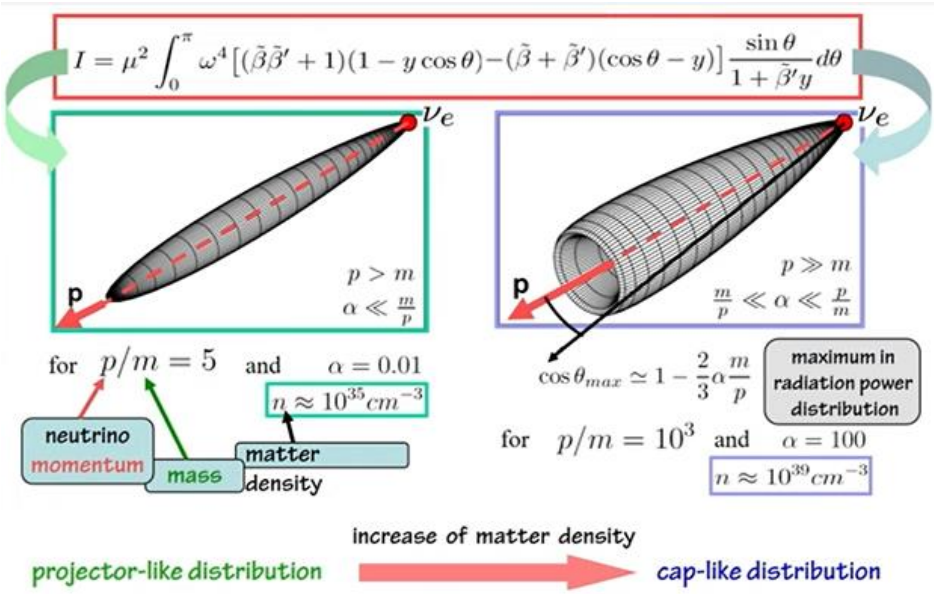
Quantum treatment of spin light of neutrino in matter shows that this process originates from the two subdivided phenomena:

1) the shift of the neutrino energy levels in the presence of the background matter, which is different for the two opposite neutrino helicity states
2) the radiation of the photon in the process of the neutrino transition from the "excited" helicity state to the low-lying helicity state in matter



Pic.14.5. Quantum theory of neutrino spin light.

Within these studies it is possible to derive the power and the rate of the neutrino spin light. It also possible to describe the special distribution of radiation power and we have found that there is a very strong projector-like effect.



Pic.14.6. Spatial distribution of radiation power.

The effect ща neutrino spin light is very slow. The characteristic time:

$$\tau = \frac{1}{\Gamma_{SL\nu}} \tag{14.47}$$

For ultra-relativistic neutrino with momentum $p \sim 10^{20}\ eV$ and magnetic moment $\mu \sim 10^{-10}\ \mu_B$ in very dense matter $n \sim 10^{40}\ cm^{-3}$ from

$$\Gamma_{SL\nu} = 4\mu^2\alpha^2 m_\nu^2 p \tag{14.48}$$

it follows that

$$\tau = \frac{1}{\Gamma_{SL\nu}} = 1.5 \times 10^{-8} s \tag{14.49}$$

In the subsequent papers we also consider the effect of plasma mass. One of our recent papers was published in the Journal of Cosmology and Astroparticle Physics about seven years ago and has a very ambitious title “Spin light of neutrino in astrophysical environments”. Indeed we examined a variety of astrophysical situations. We have found that the conditions for the neutrino spin light observation could exist in the case of short Gamma Ray Bursts.

Astrophysical bounds on neutrino magnetic moment

I would like to recall that the magnetic moment of neutrino is the most well accepted among possible neutrino electromagnetic properties. Even in the easiest generalization of the Standard Model once we accept that the mass of neutrino is not zero inevitably the magnetic moment of neutrino should be nonzero too.

First results were obtained by G.Raffelt in 1990. There are the series of papers published recent years confirming that early results. Once the magnetic moment of neutrino is not zero due to the plasmon decay to neutrino-antineutrino pair the effect of cooling of red giant stars can be modified. Using the Lagrangian

$$L_{int} = \frac{1}{2}\sum_{a,b}\left(\mu_{a,b}\bar{\psi}_a\sigma_{\mu\nu}\psi_b + \epsilon_{a,b}\bar{\psi}_a\sigma_{\mu\nu}\gamma_5\psi_b\right) \tag{14.50}$$

accounting for the magnetic moment and electric moment interaction with neutral fields the decay rate may be obtained in the following way:

$$\Gamma_{\gamma\to\nu\bar{\nu}} = \frac{\mu^2}{24\pi}\frac{(\omega^2 - k^2)^2}{\omega} \tag{14.51}$$

This magnetic moment plasma decay process enhances the Standard Model of photo-neutrino cooling of the star by proton polarization tensor. As a result faster cooling of the star could appear. In order not to be in contradictions with the observational data Raffelt obtained that the magnetic moment should be constrained:

$$\mu \leq 3 \times 10^{-12}\mu_B \tag{14.52}$$

This is the best astrophysical limit on the effective magnetic moment of neutrino. Other types of astrophysical constraints on magnetic moment of neutrino can be set. Here are three phenomena that provide the astrophysical limits:

a) the helicity change in neutrino magnetic moment scattering on particles of the environment (electrons, protons and neutrons) provides the transition from active left-handed to sterile right-handed neutrino
b) spin or spin flavor precession in transversal magnetic field $B_\perp$
c) spin or spin flavor precession in transversal matter currents $j_\perp$ or polarization $\zeta_\perp$

Applying these phenomena to concrete astrophysical settings, in particular to the experimental data on the SN 1987A, we obtain the following upper bound on neutrino magnetic moment:

$$\mu_\nu^D \sim 10^{-12}\mu_B \tag{14.53}$$

The demand of the absence of anomalous high-energy neutrinos in the flux of SN 1987A provides more or less the same upper bound.

Astrophysical bounds on neutrino millicharge

In some generalizations of the Standard Model as we have discussed on our previous lectures it is possible that neutrino is a charge particle, but this charge should be very tiny and therefore we call this electric charge of neutrino millicharge. Again using the process of plasma decay to neutrino-antineutrino pair we can set an upper bound for the neutrino millicharge. About 30 years ago by Raffelt and collaborators it was obtained that

$$q_\nu \leq 2 \times 10^{-14}e \tag{14.54}$$

From the observational data of SN 1987A neutrinos much stronger upper bound have been obtained:

$$q_\nu \leq 3 \times 10^{-17}e \tag{14.55}$$

The most severe bound comes from the demand of neutrality of neutron:

$$q_\nu \leq 3 \times 10^{-21}e \tag{14.56}$$

Together with my collaborators we obtained the most severe astrophysical bound on neutrino millicharge. We considered the general problem of neutrino motion in magnetized and rotating matter; very similar conditions can be expected in case of neutron rotating stars. As we have announced already the neutrino energy is quantized in rotating magnetized star not only due to possible neutrino electric millicharge interaction with magnetic field but also due to weak interaction of neutrinos with the rotating media. The modified Dirac equation for neutrino wave function:

$$\left\{\gamma_\mu(p^\mu + q_0 A^\mu) - \frac{1}{2}\gamma_\mu(c_l + \gamma_5)f^\mu - \frac{i}{2}\mu\sigma_{\mu\nu}F^{\mu\nu} - m\right\}\psi(x) = 0, \qquad (14.57)$$

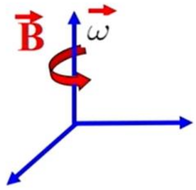
where f^μ determines the matter potential

$$V_m = \frac{1}{2}\gamma_\mu(c_l + \gamma_5)f^\mu \qquad (14.58)$$

In rotating matter

$$f^\mu = -Gn_n(1, -\epsilon y\omega, \epsilon x\omega, 0) \qquad (14.59)$$

We suppose that the axis of rotation coincide with the direction of magnetic field.



Pic.14.7. Rotation angular frequency.

It is possible to find an exact solution for neutrino wave function and also to find an exact value for neutrino energy. Neutrino momentum can be calculated according to the following formula:

$$p_0 = \sqrt{p_3^2 + 2N|2Gn_n\omega - \epsilon q_\nu B| + m^2} - Gn_n - q\phi, \qquad (14.60)$$

where p_3 — the momentum of neutrino along the fixed axis of rotation of matter and magnetic field, N — the integer number, n_n — invariant number density, ω — matter rotation frequency, q_ν — the millicharge of neutrino, B — the strength of magnetic field, ϕ — the scalar potential of electric field. Neutrino energy is quantized in rotating matter like electron energy in magnetic field (Landau energy levels):

$$p_0^{(e)} = \sqrt{m_e^2 + p_3^2 + 2\gamma N}, \gamma = eB, N = 0,1,2, \dots \qquad (14.61)$$

So the second part of the second term in (14.60) is just an effect of electric millicharge interaction with magnetic field. But there is another term that also depends on the integer number, invariant number density and matter rotation frequency. If we just switch off the electromagnetic interaction of neutrino we will get that the energy of neutrino is quantized due to its weak interaction with the rotating media.

In quasi-classical approach in presence of rotating matter neutrinos move on circular orbits. Just from this and the analogy with the Lorentz force we can introduce the generalized Lorentz force that in addition to electromagnetic part has an additional term due to weak interactions of neutrinos with the rotating media:

$$\vec{F}_{eff} = q_{eff}\vec{E}_{eff} + q_{eff}[\vec{\beta} \times \vec{B}_{eff}], \quad (14.62)$$

$$q_{eff}\vec{E}_{eff} = q_m\vec{E}_m + q_0\vec{E}, \quad (14.63)$$

$$q_{eff}\vec{E}_{eff} = |q_m\vec{B}_m + q_0\vec{B}|\vec{e}_z, \quad (14.64)$$

where $q_m = -G$ – matter induced “charge”, $\vec{E}_m = -\vec{\nabla}n_n$ – “electric” field, $\vec{B}_m = 2n_n\vec{\omega}$ – magnetic field. Low-energy neutrinos ($E \sim 1\ eV$) are trapped in circular orbits inside rotating neuron stars. We can estimate the characteristic radius; it will be less than the characteristic radius of neutron stars:

$$R = \sqrt{\frac{2N}{Gn\omega}} < R_{NS} = 10\ km, \quad (14.65)$$

where $n = 10^{37}\ cm^{-3}, \omega = 2\pi \times 10^3\ s^{-1}$. We can expect that rotating neutron stars can serve as filters for very low energy neutrinos.

I would like to return back to the most severe astrophysical bound on electric millicharge of neutrino. As we have discussed if neutrinos are millicharged particles and they are propagating in magnetized compact object the trajectories will be modified. If we consider neutrinos escaping from the rotating pulsar then they will escape the surface not perpendicular but with some inclination. A single neutrino generates a feedback force:

$$F = (q_0B + 2Gn_n\omega)\sin\theta \quad (14.66)$$

It is possible to calculate the torque produced by one single escaping neutrino due to the curved trajectory of motion of neutrino millicharge in magnetic field:

$$M_0(t) = \sqrt{1 - \frac{r^2(t)\Omega^2\sin^2\theta}{4}}\,Fr(t)\sin\theta \quad (14.67)$$

The total neutrino torque will be

$$M(t) = \frac{N_\nu}{4\pi}\int M_0(t)\sin\theta\, d\theta d\varphi \quad (14.68)$$

It should effect the initial star rotation, so the shift of the star angular velocity is

$$|\Delta\omega| = \frac{5N_\nu}{6M_s}(q_0B + 2Gn_n\omega_0), \Delta\omega = \omega - \omega_0 \quad (14.69)$$

We just introduced the so-called neutrino star turning mechanism νST. Escaping millicharged neutrinos move on curved orbits inside magnetized rotating star and feedback of effective Lorentz force should effect initial star rotation. We’ve estimated this shift of the rotation frequency in the following way:

$$\frac{|\Delta\omega|}{\omega_0} = 7.6\varepsilon \times 10^{18}\left(\frac{p_0}{10\ s}\right)\left(\frac{N_\nu}{10^{58}}\right)\left(\frac{1.4M_\odot}{M_S}\right)\left(\frac{B}{10^{14}G}\right) \quad (14.70)$$

From very conservative demand that one should avoid contradiction of νST impact with observational data on pulsars we get immediately that the millcharge of neutrino should be

$$q_0 \leq 1.3 \times 10^{-19}e_0 \quad (14.71)$$

This result is the best astrophysical bound on neutrino millcharge.

Lecture 15. Neutrino quantum states in electromagnetic fields and matter

Introduction

We've already dealt with it but I would like to attract your attention once again on the very powerful method of exact solutions and also show it's another very important applications. This method was developed within the quantum field theory describing electromagnetic interactions of particles. It is based on the use of exact wave functions of the quantum equations, for instance for electron that is the Dirac equation. We've developed this method and have applied it to the description of neutrinos which as we know have nontrivial electromagnetic properties, at least the magnetic moment of neutrino is not zero once we accept that the mass of neutrino is not zero. In other generalizations of the Standard Model neutrino can be even a millicharged particle. The neutrino charge radius is not zero. There are a set of the most important neutrino electromagnetic properties. But in addition to electromagnetic properties within the method of exact solutions we also can account for the effect of neutrino weak interaction with the background matter.

Quantum treatment of neutrino in matter

Let's consider a flavor neutrino moving in matter composed for simplicity only of electrons within the Standard Model of electroweak interactions. The Standard Model Lagrangian looks like

$$L_{int} = -\frac{g}{4\cos\theta_W}[\bar{\nu}_e\gamma^\mu(1+\gamma_5)\nu_e - \bar{e}\gamma^\mu(1-4\sin^2\theta_W+\gamma_5)e]Z_\mu \\ -\frac{g}{2\sqrt{2}}\bar{\nu}_e\gamma^\mu(1+\gamma_5)eW_\mu^+ - \frac{g}{2\sqrt{2}}\bar{e}\gamma^\mu(1+\gamma_5)\nu_e W_\mu^- \qquad (15.1)$$

There are charged current interactions and neutral current interactions contributions to neutrino potential in matter:

$$\Delta L_{eff}^{CC} = \sqrt{2}G_F\langle\bar{e}\gamma^\mu(1+\gamma_5)e\rangle\left(\bar{\nu}_e\gamma^\mu\frac{1+\gamma_5}{2}\nu_e\right), \qquad (15.2)$$

$$\Delta L_{eff}^{NC} = -\frac{G_F}{\sqrt{2}}\langle\bar{e}\gamma^\mu[(1-4\sin^2\theta_W)+\gamma_5]e\rangle\left(\bar{\nu}_e\gamma^\mu\frac{1+\gamma_5}{2}\nu_e\right) \qquad (15.3)$$

When the electron field bilinear $\langle\bar{e}\gamma^\mu(1+\gamma_5)e\rangle$ is averaged over the background

$$\langle\bar{e}\gamma_0 e\rangle \sim density, \qquad (15.4)$$

$$\langle\bar{e}\gamma_i e\rangle \sim velocity, i = 1,2,3, \qquad (15.5)$$

$$\langle\bar{e}\gamma_\mu\gamma_5 e\rangle \sim spin, \qquad (15.6)$$

it can be replaced by the matter (electrons) current that is determined by the invariant number density n and speed of matter $\vec{V}$

$$j^\mu = \left(n, n\vec{V}\right) \qquad (15.7)$$

and polarization

$$\lambda^\mu = \left(n(\vec{\zeta}\vec{V}), n\vec{\zeta}\sqrt{1-v^2} + \frac{n\vec{V}(\vec{\zeta}\vec{V})}{1+\sqrt{1-v^2}}\right), \tag{15.8}$$

$v -$ the speed of neutrino, $\vec{\zeta} -$ the spin vector of neutrino. The addition to the initial vacuum neutrino Lagrangian will be

$$\Delta L_{eff} = \Delta L_{eff}^{CC} + \Delta L_{eff}^{NC} = -f^\mu\left(\bar{\nu}\gamma_\mu\frac{1+\gamma^5}{2}\nu\right), \tag{15.9}$$

where

$$f^\mu = \frac{G_F}{\sqrt{2}}\left((1+4\sin^2\theta_W)j^\mu - \lambda^\mu\right) \tag{15.10}$$

Within the straightforward procedure we obtain the generalized Dirac equation for the neutrino wave function:

$$\left(i\gamma_\mu\partial^\mu - \frac{1}{2}\gamma_\mu(1+\gamma_5)f^\mu - m\right)\psi(x) = 0, \tag{15.11}$$

$m -$ the mass of neutrino. This equation was obtained for the first time in our paper and we also have found the exact solution for the wave function and the energy spectrum of neutrinos moving in the background matter for simplicity composed only of electrons. In the rest frame of unpolarized matter f^μ vector is simplified:

$$f^\mu = \frac{\tilde{G}_F}{2\sqrt{2}}(n,0,0,0), \tag{15.12}$$

$$\tilde{G}_F = G_F(1+4\sin^2\theta_W) \tag{15.13}$$

The equation (15.11) can be written in the Hamiltonian form:

$$i\frac{\partial}{\partial t}\psi(\vec{r},t) = \hat{H}_{matt}\psi(\vec{r},t), \tag{15.14}$$

where

$$\hat{H}_{matt} = \hat{\vec{\alpha}}\vec{p} + \hat{\beta}m + \hat{V}_{matt}, \tag{15.15}$$

$$\hat{V}_{matt} = \frac{1}{2\sqrt{2}}(1+\gamma_5)\tilde{G}_F n, \tag{15.16}$$

$$\hat{\vec{\alpha}} = \begin{pmatrix} 0 & \hat{\vec{\sigma}} \\ \hat{\vec{\sigma}} & 0 \end{pmatrix}, \tag{15.17}$$

$$\hat{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \gamma_0 \tag{15.18}$$

The form of the Hamiltonian implies that the operators of the momentum $\hat{\vec{p}}$ and the longitudinal polarization $\hat{\vec{\Sigma}}\vec{p}/p$ are the integrals of motion:

$$\frac{\hat{\vec{\Sigma}}\vec{p}}{p}\psi(\vec{r},t) = s\psi(\vec{r},t), s = \pm 1, \tag{15.19}$$

where $s = 1$ corresponds to positive-helicity state and $s = -1$ corresponds to negative-helicity state. In the relativistic limit the negative- helicity neutrino state is dominated by the left-handed chiral state: $\nu_- \approx \nu_L$, $\nu_+ \approx \nu_R$. Here are the exact solutions of the modified Dirac equation for the wave function and the energy of neutrinos for two helicity states $s = \pm 1$:

$$\psi(\vec{r},t) = e^{-i(E_\varepsilon t - \vec{p}\vec{r})}u(\vec{p},E_\varepsilon), \tag{15.20}$$

$$E_\varepsilon = \varepsilon\sqrt{\vec{p}^2\left(1 - s\alpha\frac{m}{p}\right)^2 + m^2} + \alpha m, \tag{15.21}$$

where the matter density parameter

$$\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m} \tag{15.22}$$

For instance, for the case of extreme dense matter in neutron stars when $n = 10^{37}cm^{-3}$

$$\frac{1}{2\sqrt{2}}\tilde{G}_F n \sim 1eV \tag{15.23}$$

It's very important that neutrino energy in the background matter depends on the state of the neutrino longitudinal polarization (helicity), i.e. in the relativistic case the left-handed and right-handed neutrinos with equal momenta have different energies. The exact expression for the neutrino wave function in the presence of non-moving non-polarized matter composed of electrons:

$$\psi_{\varepsilon,\vec{p},s}(\vec{r},t) = \frac{e^{-i(E_\varepsilon t - \vec{p}\vec{r})}}{2L^{3/2}}\begin{pmatrix} \sqrt{1 + \frac{m}{E_\varepsilon - \alpha m}}\sqrt{1 + s\frac{p_3}{p}} \\ s\sqrt{1 + \frac{m}{E_\varepsilon - \alpha m}}\sqrt{1 - s\frac{p_3}{p}}e^{i\delta} \\ s\varepsilon\eta\sqrt{1 - \frac{m}{E_\varepsilon - \alpha m}}\sqrt{1 + s\frac{p_3}{p}} \\ \varepsilon\eta\sqrt{1 - \frac{m}{E_\varepsilon - \alpha m}}\sqrt{1 - s\frac{p_3}{p}}e^{i\delta} \end{pmatrix}, \tag{15.24}$$

$$\eta = \text{sign}\left(1 - s\alpha\frac{m}{p}\right), \tag{15.25}$$

$$\delta = \arctan(p_2/p_1), \tag{15.26}$$

$$E_\varepsilon - \alpha m = \varepsilon\sqrt{\vec{p}^2\left(1 - s\alpha\frac{m}{p}\right)^2 + m^2} \tag{15.27}$$

The quantity $\varepsilon = \pm 1$ splits the solutions into two branches that in the limit of vanishing matter density $\alpha \to 0$ reproduce the positive and negative-frequency solutions, respectively.

It is also possible to apply the discussed procedure to the case when neutrinos are moving in matter composed not only of electrons but also of protons and neutrons. There is the same modified Dirac equation (15.11) with

$$f^\mu = \sqrt{2}G_F\sum_{f=e,p,n} j_f^\mu q_f^{(1)} + \lambda_f^\mu q_f^{(2)}, \tag{15.28}$$

$$q_f^{(1)} = \left(I_{3L}^{(f)} - 2Q^{(f)}\sin^2\theta_W + \delta_{ef}\right), \tag{15.29}$$

$$q_f^{(2)} = -\left(I_{3L}^{(f)} + \delta_{ef}\right), \tag{15.30}$$

$$\delta_{ef} = \begin{cases} 1 & for\ f = e \\ 0 & for\ f = n, p \end{cases} \tag{15.31}$$

Neutrino energy spectrum in matter composed of electrons, protons and neutrons:

$$E_\varepsilon = \varepsilon\sqrt{\vec{p}^2\left(1 - s\alpha\frac{m}{p}\right)^2 + m^2} + \alpha m, \tag{15.32}$$

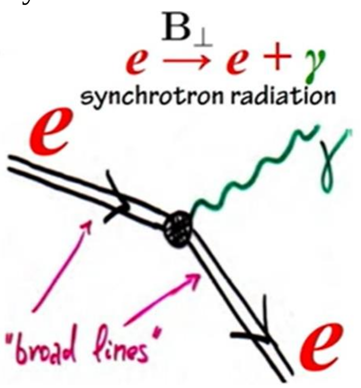
where $\varepsilon = \pm 1, s = \pm 1$ and the density parameter α is determined by particle's number densities $n_{e,p,n}$. For instance, for electron neutrino

$$\alpha_{\nu_e} = \frac{1}{2\sqrt{2}}\frac{G_F}{m}\left(n_e(1 + 4\sin^2\theta_W) + n_p(1 - 4\sin^2\theta_W) - n_n\right) \tag{15.33}$$

In electrically neutral $n_e = n_p$ and neutron reach matter $n_n \gg n_e, n_p$ we obtain that $\alpha_{\nu_e} < 0$. For electron antineutrino $\alpha_{\bar{\nu}_e} \to -\alpha_{\nu_e}$. For muon and tau neutrino

$$\alpha_{\nu_\mu,\nu_\tau} = \frac{1}{2\sqrt{2}}\frac{G_F}{m}\left(n_e(4\sin^2\theta_W - 1) + n_p(1 - 4\sin^2\theta_W) - n_n\right) \tag{15.34}$$

It's very important that the method that we are developing is similar to the well-known Furry representation of quantum electrodynamics.



Pic.15.1. Synchrotron radiation.

The method of exact solutions was developed for application to the theory of synchrotron radiation. According to the method of exact solutions for the initial and final electron states we use the exact solutions for the Dirac equation in electromagnetic field. The Dirac equation for electron wave function accounting for interaction with electromagnetic field:

$$\left\{\gamma^\mu\left(i\partial_\mu - eA_\mu^{ext}(x)\right) - m_e\right\}\psi_F(x) = 0, \tag{15.35}$$

where electromagnetic field potential

$$A_\mu(x) = A_\mu^q(x) + A_\mu^{ext}(x) \tag{15.36}$$

I would like to compare neutrino and antineutrino energy spectra in matter. For the fixed value of the neutrino momentum $\vec{p}$ there are two values for the “positive sign” $\varepsilon = +1$ energies:

– *positive-helicity neutrino energy*

$$E^{s=+1} = \sqrt{\vec{p}^2\left(1-\alpha\frac{m}{p}\right)^2 + m^2} + \alpha m, \tag{15.37}$$

– *negative-helicity neutrino energy*

$$E^{s=-1} = \sqrt{\vec{p}^2\left(1+\alpha\frac{m}{p}\right)^2 + m^2} + \alpha m, \tag{15.38}$$

So the energies in matter depend on the helicity in contrary to the case of vacuum when the matter density perimeter is equal to zero. The two other values of the energy for the “negative sign” $\varepsilon = -1$ correspond to the antiparticle solutions. By changing the sign of the energy, we obtain

– *positive-helicity antineutrino energy*

$$\tilde{E}^{s=+1} = \sqrt{\vec{p}^2\left(1-\alpha\frac{m}{p}\right)^2 + m^2} - \alpha m, \tag{15.39}$$

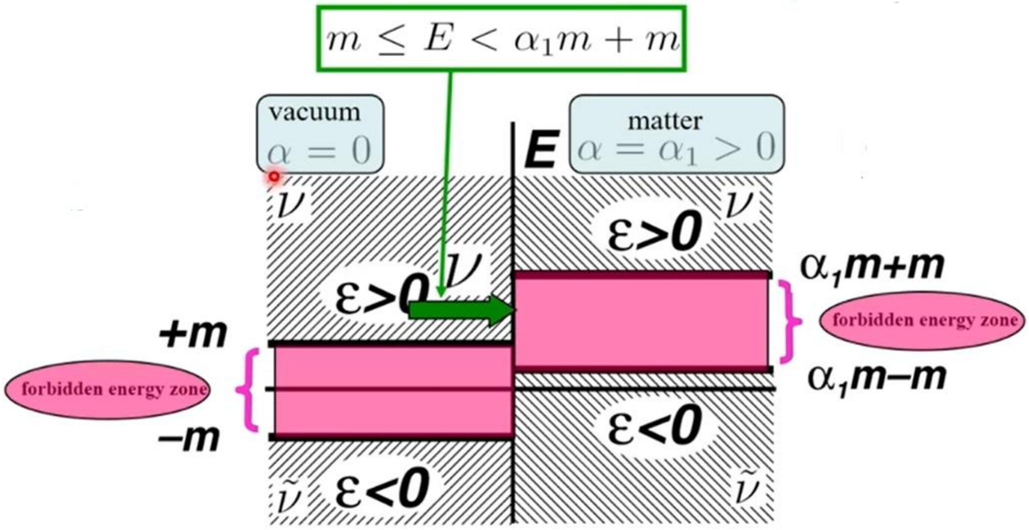
– *negative-helicity antineutrino energy*

$$\tilde{E}^{s=-1} = \sqrt{\vec{p}^2\left(1+\alpha\frac{m}{p}\right)^2 + m^2} - \alpha m, \tag{15.40}$$

Now, after the introduction on the solutions of the modified equation in presence of matter for neutrinos and antineutrinos with different helicities, I would like to comment on some quite straightforward new effects that can be easily understood using the exact solutions:

- Neutrino reflection from interface between vacuum and matter
- Neutrino trapping in matter
- Neutrino-antineutrino pair annihilation at interface between vacuum and matter
- Spontaneous neutrino-antineutrino pair creation in matter

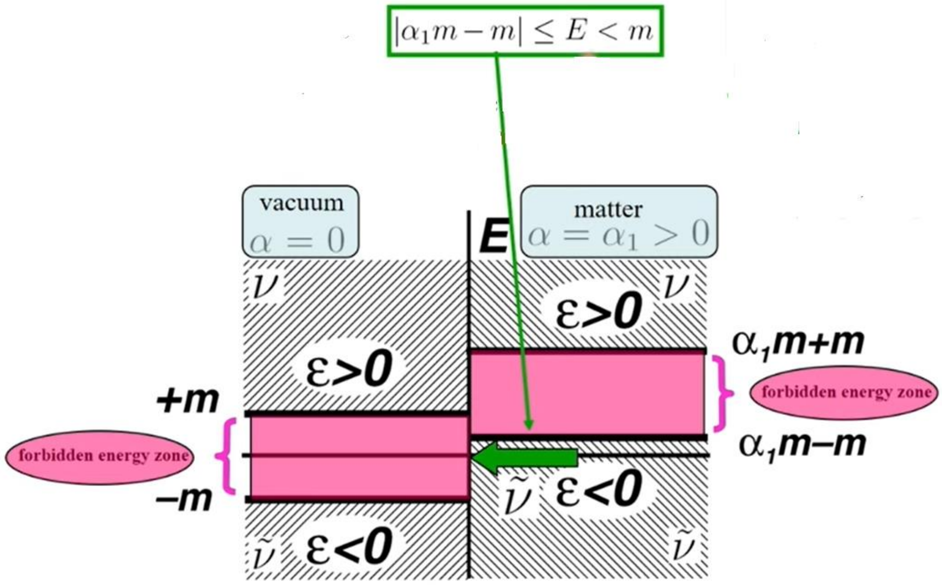
Neutrino reflection from interface between vacuum and matter



Pic.15.2. Neutrino reflection from interface between vacuum and matter.

Let's suppose that there is a surface that splits space into two parts. The left-hand side corresponds to the vacuum with zero matter density parameter $\alpha = 0$. The right-hand side of space is occupied by matter with $\alpha = \alpha_1 > 0$. From the exact expressions for the energies in vacuum known before our discussion and in matter as we have derived just in the previous paragraph we see that there is quite complicated structure of available ranges of energies for neutrino. If the neutrino energy in vacuum E is less than the neutrino minimal energy in medium $\alpha_1 m + m$ then the appropriate energy level inside the medium is not accessible for neutrino and neutrino is reflected from the interface. This effect exist because the forbidden zones in vacuum and in matter are not identical, they are shifted.

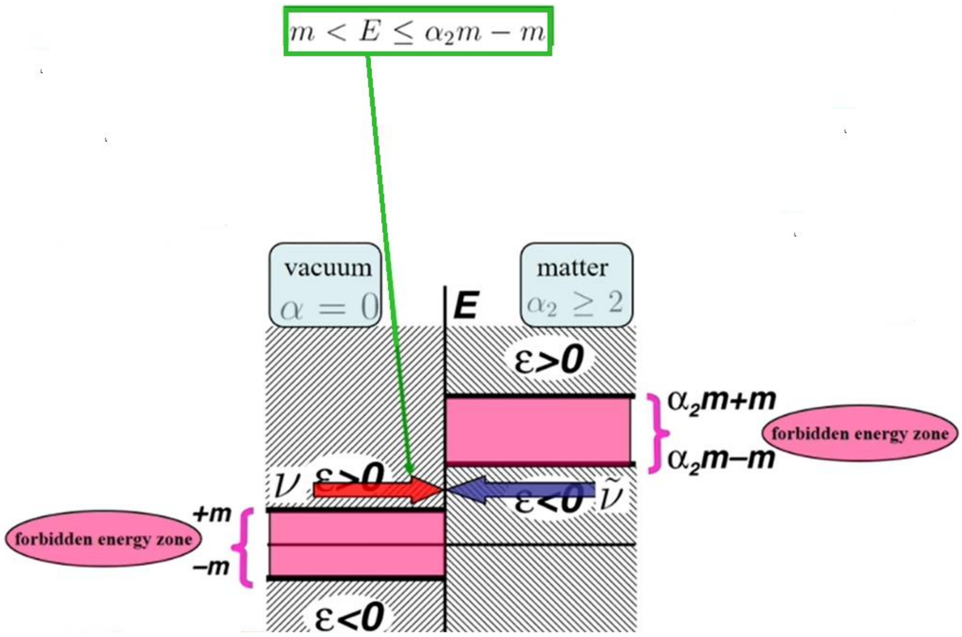
Neutrino trapping in matter



Pic.15.3. Neutrino trapping in matter

Let's consider antineutrino moving in medium with the energy within the following range $|\alpha_1 m - m| \leq E < m$. It cannot escape from the medium because this particular range of energies exactly falls on the forbidden energy zone in vacuum. The antineutrino has not enough energy to survive in vacuum; it is trapped inside the medium.

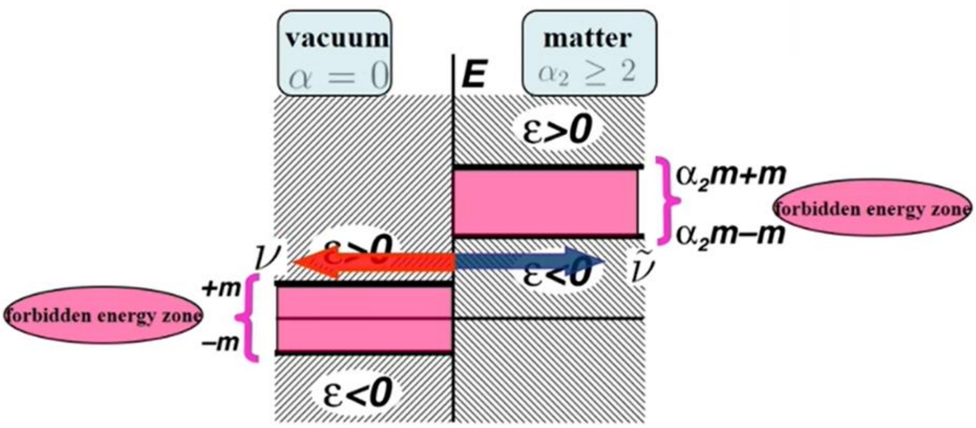
Neutrino-antineutrino pair annihilation at interface between vacuum and matter



Pic.15.4. Neutrino-antineutrino pair annihilation at interface between vacuum and matter.

Let's consider neutrino with energy $m < E \leq \alpha_2 m - m$ propagating in vacuum towards the interface with matter, where the matter dense parameter $\alpha_2 \geq 2$. If not all of “negative sign” energy levels are occupied and, in particular, the level with energy exactly equal to E is available let’s suppose an antineutrino exists in matter. There is a possibility for neutrino-antineutrino annihilation $\nu + \tilde{\nu} \to \gamma$ at the interface of vacuum and matter.

Spontaneous neutrino-antineutrino pair creation in matter



Pic.15.5. Spontaneous neutrino-antineutrino pair creation in matter

Again we suppose that there is a gap between the forbidden energy zones for neutrinos in vacuum and in matter, it happens when the matter density parameter $\alpha_2 \geq 2$. Neutrino-antineutrino pair creation can be interpreted as a process of appearance of a particle state in the “positive sign” energy range accompanied by appearance of the hole state in the “negative sign” energy sea. This effect recalls the well-known spontaneous electron-positron pair creation according to the Klein’s paradox of electrodynamics.

I would like to make one additional important note. We have obtained the following expression for the energy spectrum of active left-handed and sterile right-handed neutrino:

$$E = \sqrt{\vec{p}^2 \left(1 - s\alpha \frac{m}{p}\right)^2 + m^2 + \alpha m} \qquad (15.41)$$

If we consider ultra-relativistic case when the momentum exceed the mass of neutrino $p \gg m$ and also the density is not extremely high

$$\alpha = \frac{1}{2\sqrt{2}} \tilde{G}_F \frac{n}{m} \ll \frac{p}{m} \qquad (15.42)$$

then from the obtained neutrino energy spectrum (15.41) we immediately get the approximation for the two helicity states neutrino energies:

$$E^{s=\pm 1} \approx E_0 - \alpha m \left(s \frac{p}{E_0} - 1\right) \qquad (15.43)$$

The energies (15.43) reproduce by the way in the case of relativistic neutrinos the exact expressions for the energies of neutrino chiral states:

– *active left-handed neutrino energy*

$$E_{\nu_L} \approx E^{s=-1} \approx E_0 + \frac{1}{\sqrt{2}}\tilde{G}_F n, \tag{15.44}$$

– *sterile left-handed neutrino energy*

$$E_{\nu_R} \approx E^{s=+1} \approx E_0 \tag{15.45}$$

Neutrino flavor oscillations in matter

Let's consider two neutrino flavor states ν_e and ν_μ that propagate in electrically neutral matter $n_e = n_p$ composed of neutrons, electrons and protons in equal number. From the previous paragraphs we get the following matter density parameters:

$$\alpha_{\nu_e} = \frac{1}{2\sqrt{2}}\frac{G_F}{m}\left(n_e(1 + 4\sin^2\theta_W) + n_p(1 - 4\sin^2\theta_W) - n_n\right), \tag{15.46}$$

$$\alpha_{\nu_\mu,\nu_\tau} = \frac{1}{2\sqrt{2}}\frac{G_F}{m}\left(n_e(4\sin^2\theta_W - 1) + n_p(1 - 4\sin^2\theta_W) - n_n\right) \tag{15.47}$$

For the relativistic active neutrinos we get the very simplified energy expression:

$$E^{s=-1}_{\nu_e,\nu_\mu} \approx E_0 + 2\alpha_{\nu_e,\nu_\mu} m_{\nu_e,\nu_\mu} \tag{15.48}$$

The difference between active electron and muon neutrino energies:

$$\Delta E \equiv E^{s=-1}_{\nu_e} - E^{s=-1}_{\nu_\mu} = \sqrt{2} G_F n_e \tag{15.49}$$

From (15.49) follows the well-known expression for the MSW resonance effect.

Modified Dirac-Pauli equation for neutrino in matter

We have developed the quasi-classical approach to a massive neutrino spin evolution in the presence of external electromagnetic fields $F_{\mu\nu}$ and background matter. The well-known Bargmann-Michel-Telegdi equation of QED has been generalized for the case of a neutrino moving in matter and external electromagnetic fields by the following substitution of the electromagnetic field tensor:

$$F_{\mu\nu} \to E_{\mu\nu} = F_{\mu\nu} + G_{\mu\nu}, \tag{15.50}$$

where $G_{\mu\nu}$ accounts for the presence of matter

$$G^{\mu\nu} = \sum_f \epsilon^{\mu\nu\eta\lambda}\left(\rho_f^{(1)} j_\eta^f + \rho_f^{(2)} \lambda_\eta^f\right) u_\lambda, \tag{15.51}$$

$$\rho_e^{(1)} = \frac{G_F}{2\mu\sqrt{2}}(1 + 4\sin^2\theta_W), \tag{15.52}$$

$$\rho_e^{(2)} = -\frac{G_F}{2\mu\sqrt{2}}, \tag{15.53}$$

$f = e, n, p, \mu, \tau, \ldots$, $j_\eta^f -$ matter current, $\lambda_\eta^f -$ matter polarization, $u_\lambda -$ neutrino speed. The substitution (15.50) means that we shift the real magnetic and electric fields:

$$\vec{B} = \vec{B} + \vec{M}, \tag{15.54}$$

$$\vec{E} = \vec{E} - \vec{P} \tag{15.55}$$

We've obtained the modified Bargmann-Michel-Telegdi equation for neutrino spin evolution with the second term accounting for weak interactions of neutrinos with the background matter:

$$\frac{d\vec{S}_\nu}{dt} = 2\mu_\nu[\vec{S}_\nu \times \vec{B}] + 2\mu_\nu[\vec{S}_\nu \times \vec{G}] \tag{15.56}$$

The more complete treatment of electromagnetic interactions of course is provided by the Dirac-Pauli quantum equation that can be derived from the Dirac-Schwinger equation:

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = \int M_F(x', x)\psi(x')dx', \tag{15.57}$$

$M_F(x', x) -$ neutrino mass operator in electromagnetic field. In the linear approximation over the interaction of electron with the external electromagnetic field one can easily obtain the well-known Dirac-Pauli equation:

$$\left(i\gamma^\mu \partial_\mu - m - \frac{\mu}{2}\sigma^{\mu\nu}F_{\mu\nu}\right)\psi(x) = 0, \tag{15.58}$$

We have proceeded the analysis procedure and generalized this equation accounting not only for the electromagnetic interaction of neutrinos but also for the neutrino interaction with the background matter. Using the substitution $F_{\mu\nu} \to G_{\mu\nu}$ from (15.58) we obtain

$$\left(i\gamma^\mu \partial_\mu - m - \frac{\mu}{2}\sigma^{\mu\nu}G_{\mu\nu}\right)\psi(x) = 0, \tag{15.59}$$

$$G^{\mu\nu} = \gamma\rho^{(1)}n \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta_3 & \beta_2 \\ 0 & \beta_3 & 0 & -\beta_1 \\ 0 & -\beta_2 & \beta_1 & 0 \end{pmatrix}, \tag{15.60}$$

$$\rho_e^{(1)} = \frac{\tilde{G}_F}{2\sqrt{2}\mu}, \gamma = \left(1 - \vec{\beta}^2\right)^{-1/2}, \vec{\beta} = (\beta_1, \beta_2, \beta_3), \tag{15.61}$$

$\vec{\beta} -$ neutrino speed. It is possible to find the stationary states for this equation. Neutrino wave function in matter:

$$\psi(\vec{r}, t) = e^{-i(Et - \vec{p}\vec{r})}u(\vec{p}, E) \tag{15.62}$$

Neutrino energy spectrum in matter for two helicity states $s = \pm 1$:

$$E = \sqrt{\vec{p}^2(1 + \alpha^2) + m^2 - 2\alpha mps}, \tag{15.63}$$

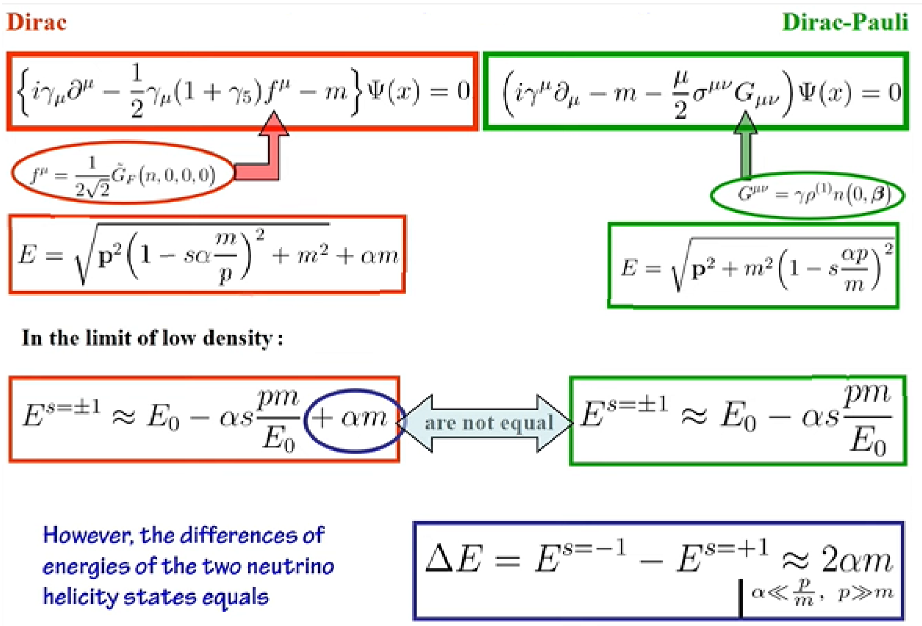
where the matter density parameter

$$\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m} \tag{15.64}$$

The exact expression for the neutrino wave function:

$$\psi_{\vec{p},s}(\vec{r},t)=\frac{e^{-i(Et-\vec{p}\vec{r})}}{2L^{3/2}}\begin{pmatrix}\sqrt{1+\frac{m-s\alpha p}{E}}\sqrt{1+s\frac{p_3}{p}}\\ s\sqrt{1+\frac{m-s\alpha p}{E}}\sqrt{1-s\frac{p_3}{p}}e^{i\delta}\\ s\varepsilon\eta\sqrt{1-\frac{m-s\alpha p}{E}}\sqrt{1+s\frac{p_3}{p}}\\ \varepsilon\eta\sqrt{1-\frac{m-s\alpha p}{E}}\sqrt{1-s\frac{p_3}{p}}e^{i\delta}\end{pmatrix} \tag{15.65}$$

We have two identical equations and the corresponding solutions accounting for neutrino interaction with the background matter. One is based on the modified Dirac equation, another - on Dirac-Pauli equation.



Pic.15.6. Two energy spectra.

We see that the energies are different but if we consider the limit of low density then this difference will be explicitly demonstrated by the additional term αm. The difference of energies of two neutrino helicity states calculated using the energy spectrum of the modified Dirac equation and calculated using the energy spectrum of the Dirac-Pauli equation are the same. It means that these energy spectrums provide the identical expressions for the probability of MSW resonance amplification of flavor oscillations.

Accounting simultaneously for interactions with external electromagnetic fields and also for weak interaction with background matter we derived the following modified Dirac-Pauli equation:

$$\left(i\gamma^{\mu}\partial_{\mu} - m - \frac{\mu}{2}\sigma^{\mu\nu}\left(G_{\mu\nu} + F_{\mu\nu}\right)\right)\psi(x) = 0 \qquad (15.66)$$

If constant magnetic field is present in the background and a neutrino is moving parallel (or anti-parallel) to the field vector $\vec{B}$, then the neutrino energy spectrum can be obtained by

$$\alpha \to \alpha' = \alpha + \frac{\mu B_{||}}{p} \Rightarrow \qquad (15.67)$$

$$E = \sqrt{\vec{p}^2 + m^2\left(1 - s\frac{\alpha p + \mu B_{||}}{m}\right)^2} \qquad (15.68)$$

It is also possible to calculate the gap between the two neutrino helicity states in magnetized matter:

$$\Delta_{eff} = E^{s=-1} - E^{s=+1} = \frac{\tilde{G}_F}{\sqrt{2}}n + 2\frac{\mu B_{||}}{\gamma} \qquad (15.69)$$

The additional term here is proportional to the neutrino magnetic moment interaction with the longitudinal magnetic field; however this term is strongly suppressed by gamma-factor of neutrino.

Neutrino oscillations in magnetized matter

The Dirac-Pauli energy spectrum of neutrino in magnetized matter can be used for derivation of the neutrino oscillation $\nu_L \leftrightarrow \nu_R$ probability:

$$P_{\nu_L \to \nu_R}(x) = \sin^2 2\theta_{eff} \sin^2 \frac{\pi x}{L_{eff}}, \qquad (15.70)$$

where

$$\sin^2 2\theta_{eff} = \frac{E_{eff}^2}{E_{eff}^2 + \Delta_{eff}^2}, E_{eff} = 2\mu B_{\perp}, \qquad (15.71)$$

$$L_{eff} = \frac{2\pi}{\sqrt{E_{eff}^2 + \Delta_{eff}^2}}, \qquad (15.72)$$

$$\Delta_{eff} = E^{s=-1} - E^{s=+1} = \frac{\tilde{G}_F}{\sqrt{2}}n + 2\frac{\mu B_{||}}{\gamma} \qquad (15.73)$$

Just for completeness I would like to mention that it is also possible to solve the problem of neutrino propagation in matter using the method of exact solutions. The equation for neutrino Green function in matter:

$$\left(i\gamma_{\mu}\partial^{\mu} - \frac{1}{2}\gamma_{\mu}(1 + \gamma_5)f^{\mu} - m\right)G(x) = -\delta(x) \qquad (15.74)$$

Again the matter effect is described by f^{μ} vector that is determined by the matter current and polarization:

$$f^\mu = \frac{G_F}{\sqrt{2}}\left((1+4\sin^2\theta_W)j^\mu - \lambda^\mu\right) \tag{15.75}$$

In the momentum representation (15.74) looks like the following

$$\left(\hat{p} - m - \hat{f}P_L\right)G(p) = -1, \tag{15.76}$$

where $P_L = \frac{1+\gamma_5}{2}, P_R = \frac{1-\gamma_5}{2}$. It is possible to find an exact solution for the neutrino Green function:

$$G_{matt}(p) = \\ = \frac{-(p^2-m^2)(\hat{p}+m) + \hat{f}(\hat{p}-m)P_L(\hat{p}+m) - f^2\hat{p}P_L + 2(fp)P_R(\hat{p}+m)}{(p^2-m^2)^2 - 2(fp)(p^2-m^2) + f^2p^2} \tag{15.77}$$

I would like to specify some new effects in neutrino spin and spin-flavor oscillations that can provide new important consequences for neutrinos in different extreme astrophysical environments. First of all, I would like to speak about generation of neutrino spin $\nu_e^L \leftarrow (j_\perp) \rightarrow \nu_e^R$ and spin-flavor $\nu_e^L \leftarrow (j_\perp) \rightarrow \nu_\mu^R$ oscillations by interaction with nontrivial transversal matter current $j_\perp$. Our discussion mostly will be based on the paper published by me with Pavel Pustoshny in 2018, but this effect as I have already mentioned in previous lectures was predicted in my paper in 2004. The second important phenomenon is the inherent interplay of neutrino spin and spin-flavor oscillations in magnetic field B. This discussion mostly will be based on our paper published by me with Anton Popov.

Neutrino spin and spin-flavor oscillations due to transversal matter current

Let's start with the first effect. There is no need for either magnetic moment of neutrino or magnetic field. In the conclusion of my paper (2004) one can read the following statement: "The possible emergence of neutrino spin oscillations (for example, $\nu_{e_L} \leftrightarrow \nu_{e_R}$) owing to neutrino interaction with matter under the condition that there exists a nonzero transverse current component or matter polarization (that is, $\vec{M}_{0_\perp} \neq 0$) is the most important new effect that follows from the investigation of neutrino spin oscillations in Section 4. So far, it has been assumed that neutrino spin oscillations may arise only in the case where there exists a nonzero transverse magnetic field in the neutrino rest frame." I have also derived the probability of spin $\nu_{e_L} \rightarrow \nu_{e_R}$ and spin-flavor $\nu_{e_L} \rightarrow \nu_{\mu_R}$ oscillations in this paper:

$$P\left(\nu_i \rightarrow \nu_j\right) = \sin^2 2\theta_{eff} \sin^2\left(\frac{\pi x}{L_{eff}}\right), i \neq j, \tag{15.78}$$

where

$$L_{eff} = \frac{2\pi}{\sqrt{E_{eff}^2 + \Delta_{eff}^2}}, \tag{15.79}$$

$$\sin^2 2\theta_{eff} = \frac{E_{eff}^2}{E_{eff}^2 + \Delta_{eff}^2}, \tag{15.80}$$

$$\Delta^2_{eff} = \frac{\mu}{\gamma_\nu}\left|\vec{B}_{0_{||}} + \vec{M}_{0_{||}}\right|, \tag{15.81}$$

$$E_{eff} = \mu\left|\vec{B}_{\perp} + \frac{1}{\gamma_\nu}\vec{M}_{0_{\perp}}\right| \tag{15.82}$$

From these formulas it follows that even in the absence of magnetic field and magnetic moment there still will be one contribution due to transversal component $\vec{M}_{0_\perp}$ given by the second term in

$$\vec{M}_0 = \gamma_\nu \rho n_e \left(\vec{\beta}_\nu\left(1 - \vec{\beta}_\nu \vec{v}_e\right) - \frac{1}{\gamma_\nu}\vec{v}_{e_\perp}\right), \tag{15.83}$$

where n_e – the matter density, $\gamma_\nu = E_\nu/m_\nu$ – the Lorentz factor of neutrino and

$$\rho = \frac{G_F}{2\mu_\nu\sqrt{2}}\left(1 + 4\sin^2\theta_W\right) \tag{15.84}$$

This effect of neutrino helicity conversions and oscillations induced by transversal matter current has been recently confirmed in studies of neutrino propagation in astrophysical media independently by other people. In our paper (2018) we consider spin and spin-flavor oscillations within quantum treatment. Firstly, we introduce two flavor states and for each of the flavor state two possible spin orientations

$$\nu_f = \left(\nu_e^+, \nu_e^-, \nu_\mu^+, \nu_\mu^-\right)^T \tag{15.85}$$

We account for neutrino interactions with the longitudinal $\vec{j}_{||}$ and transversal $\vec{j}_\perp$ matter currents. For simplicity we suppose that matter is composed of neutrons, neutron number density in laboratory frame:

$$n = \frac{n_0}{\sqrt{1 - v^2}}, \tag{15.86}$$

$\vec{v} = (v_1, v_2, v_3)$ – the velocity of matter. The addition to the Lagrangian accounting for two flavor neutrino interactions with the particular background of neutrons:

$$L_{int} = -f^\mu \sum_{l=e,\mu} \bar{\nu}_l(x)\gamma_\mu \frac{1+\gamma_5}{2}\nu_l(x) = -f^\mu \sum_{i=1,2} \bar{\nu}_i(x)\gamma_\mu \frac{1+\gamma_5}{2}\nu_i(x), \tag{15.87}$$

$$f^\mu = -\frac{G_F}{2\sqrt{2}} j_n^\mu, \tag{15.88}$$

$$j_n^\mu = n(1, \vec{v}) \tag{15.89}$$

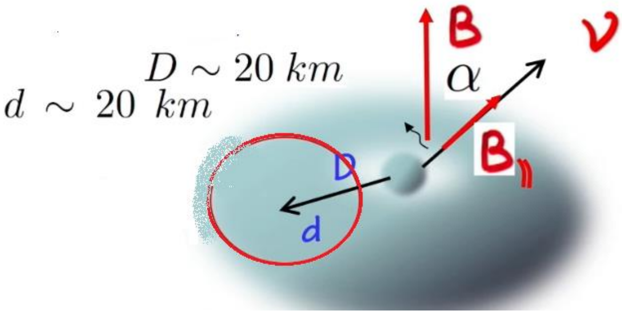
We also included the effect of possible neutrino interaction with the magnetic field. We have derived the evolution equation:

$$i\frac{d}{dt}\nu_f^s = \left(H_0 + \Delta H_0^{SM} + \Delta H_{j_{||}+j_\perp}^{SM} + \Delta H_{B_{||}+B_\perp}^{SM} + \Delta H_0^{NSI} + \Delta H_{j_{||}+j_\perp}^{NSI}\right)\nu_f^s, \tag{15.90}$$

where the first part of the effective evolution Hamiltonian corresponds to vacuum $H_0 \to$ vacuum and $\Delta H_0^{SM} \to$ Standard Model interaction with matter at rest, $\Delta H_{j_{||}+j_\perp}^{SM} \to$ Standard Model interaction with moving matter, $\Delta H_{B_{||}+B_\perp}^{SM} \to$ Standard Model interaction with magnetic field, ΔH_0^{NSI} → Non-Standard Interactions with matter at rest, $\Delta H_{j_{||}+j_\perp}^{NSI}$ → Non-Standard Interactions with moving matter. We've solved this equation and found the solution for the probabilities for different cases. We examined the possibility of resonance amplification of neutrino oscillations in several particular cases:

- neutrino spin oscillations $\nu_e^L \leftarrow (j_\perp) \rightarrow \nu_e^R$ engaged by the transversal matter current and it's resonance amplification due to neutrino interaction with the longitudinal matter current $j_{||}$
- neutrino spin oscillations $\nu_e^L \leftarrow (j_\perp) \rightarrow \nu_e^R$ engaged by the transversal matter current and it's resonance amplification due to neutrino interaction with the longitudinal magnetic field $B_{||}$
- neutrino spin-flavor oscillations $\nu_e^L \leftarrow (j_\perp) \rightarrow \nu_\mu^R$ engaged by the transversal matter current and it's resonance amplification due to matter-at-rest effect
- neutrino spin-flavor oscillations $\nu_e^L \leftarrow (j_\perp^{NSI}) \rightarrow \nu_\mu^R$ engaged by the Non-Standard Interactions of neutrino with the transversal matter current and it's resonance amplification due to matter-at-rest effect

We considered the neutrino spin oscillations $\nu_e^L \leftarrow (j_\perp) \rightarrow \nu_e^R$ induced by the transversal matter current and we applied our theoretical model to the concrete model of the short Gamma Ray Burst following the paper of Perego of 2014. By the way the same model we used in our paper examining the particular astrophysical environments where is the spin light of neutrino might be important. So, in our paper we considered neutrino escaping the central neutron star with some inclination angle α from accretion disc.



Pic.15.7. Neutrino escaping the central neutron star.

It is possible to estimate the longitudinal part of magnetic field $B_{||} = B\sin\alpha \sim \frac{1}{2}B$. We suppose that the toroidal bulk rotates with the frequency of $\omega = 10^3 s^{-1}$. It generates the transversal velocity of matter in respect to indicated neutrino propagation $v_\perp = \omega D = 0.067$ and $\gamma_n = 1.002$. We estimated the effective energy and the other very important quantity that contributes to the oscillation probability:

$$E_{eff} = \left(\frac{\eta}{\gamma}\right)_{ee} \tilde{G} n v_\perp = \frac{\cos^2\theta}{\gamma_{11}} \tilde{G} n v_\perp \approx \tilde{G} n_0 \frac{\gamma_n}{\gamma_\nu} v_\perp, \tag{15.91}$$

$$\Delta_{eff} = \left| \left(\frac{\mu}{\gamma}\right)_{ee} \vec{B}_{||} + \eta_{ee} \tilde{G} n \vec{\beta} \right| \approx \left| \frac{\mu_{11}}{\gamma_\nu} B_{||} - \tilde{G} n_0 \gamma_n \right| \tag{15.92}$$

According to (15.80) the resonance is reached when Δ_{eff} vanishing and it is means that the resonance condition looks like the following

$$E_{eff} \geq \Delta_{eff} \tag{15.93}$$

It will be fulfilled when

$$\left| \frac{\mu_{11} B_{||}}{\tilde{G} n_0 \gamma_n} - \gamma_\nu \right| \leq 1 \tag{15.94}$$

In other case we considered the neutrino spin-flavor oscillations $\nu_e^L \leftarrow (j_\perp, B_\perp) \rightarrow \nu_\mu^R$ induced by the transversal matter current and transversal magnetic field. The resonance amplification of spin-flavor oscillations criterion is

$$\sin^2 2\theta_{eff} = \frac{E_{eff}^2}{E_{eff}^2 + \Delta_{eff}^2} \geq \frac{1}{2} \tag{15.95}$$

We've derived the effective energy:

$$E_{eff} = \left| \mu_{e\mu} B_\perp + \left(\frac{\eta}{\gamma}\right)_{e\mu} \tilde{G} n v_\perp \right| \geq \left| \Delta M - \frac{1}{2}\left(\frac{\mu_{11}}{\gamma_{11}} + \frac{\mu_{22}}{\gamma_{22}}\right) B_{||} - \tilde{G} n \left(1 - \vec{v}\vec{\beta}\right) \right| \tag{15.96}$$

For the particular choice of the mass square difference $\Delta m^2 = 7.37 \times 10^{-5}\ eV^2$, $\sin^2\theta = 0.297$ and the realistic energies of neutrinos $p_0^\nu = 10^6\ eV$ we have found that the resonance of these spin-flavor oscillations will be realized when matter has quite reasonable density about $n_0 \approx 5 \times 10^{36}\ cm^{-3}$. For this particular choice of parameters the effective oscillations length will also get quite reasonable value of about $L_{eff} \approx 10\ km$.

Neutrino spin and spin-flavor oscillations in constant magnetic field

Now I would like to consider neutrino eigenstates and flavor $\nu_e^L \leftrightarrow \nu_\mu^L$, spin $\nu_e^L \leftrightarrow \nu_e^R$ and spin-flavor $\nu_e^L \leftrightarrow \nu_\mu^R$ oscillations in constant magnetic field. So we consider two flavor states with two different helicities. Each of the flavor helicity states can be expressed in terms of the mass states $\nu_i^{L(R)}$:

$$\begin{cases} \nu_e^{L(R)} = \nu_1^{L(R)} \cos\theta + \nu_2^{L(R)} \sin\theta \\ \nu_\mu^{L(R)} = -\nu_1^{L(R)} \sin\theta + \nu_2^{L(R)} \cos\theta \end{cases} \tag{15.97}$$

We also distinguished the transversal and longitudinal components of constant magnetic field $\vec{B} = \left(B_\perp, 0, B_{||}\right)$. We've derived the Dirac equation:

$$\left(\gamma_\mu p^\mu - m_i - \mu_i \vec{\Sigma}\vec{B}\right) \nu_i^s(p) = 0 \tag{15.98}$$

and also examined the stationary states of neutrinos in presence of magnetic field:

$$\begin{cases} \nu_i^L(t) = c_i^+ \nu_i^+(t) + c_i^- \nu_i^-(t) \\ \nu_i^R(t) = d_i^+ \nu_i^+(t) + d_i^- \nu_i^-(t) \end{cases} \tag{15.99}$$

The neutrino spin properties are characterized by the spin operator

$$\hat{S}_i = \frac{1}{N}\left[\vec{\Sigma}\vec{B} - \frac{i}{m_i}\gamma_0\gamma_5\left[\vec{\Sigma} \times \vec{p}\right]\vec{B}\right], \tag{15.100}$$

$$\frac{1}{N}=\frac{m_i}{\sqrt{m_i^2\vec{B}^2+\vec{p}^2B_\perp^2}} \quad (15.101)$$

that commutes with the evolution Hamiltonian:

$$\widehat{H}_i=\gamma_0\vec{\gamma}\vec{p}-\mu_i\gamma_0\vec{\Sigma}\vec{B}+m_i\gamma_0 \quad (15.102)$$

The energy spectrum of neutrinos is shown below:

$$E_i^s=\sqrt{m_i^2+p^2+\mu_i^2\vec{B}^2+2\mu_i s\sqrt{m_i^2\vec{B}^2+\vec{p}^2B_\perp^2}} \quad (15.103)$$

By the way, this expression is not new, it was found for the first time many years ago when people considered a neutron propagation in presence of constant magnetic field. For simplicity we introduced new notations:

$$\mu_\pm=\frac{1}{2}(\mu_1\pm\mu_2), \quad (15.104)$$

μ_1,μ_2 – the magnetic moment of neutrino mass states. Using the exact solutions we calculated the probabilities of

– *flavor oscillations*

$$P_{\nu_e^L\to\nu_\mu^L}(t)=\sin^2 2\theta\left\{\cos(\mu_1B_\perp t)\cos(\mu_2B_\perp t)\sin^2\frac{\Delta m^2}{4p}t\right. \\ \left.+\sin^2(\mu_+B_\perp t)\sin^2(\mu_-B_\perp t)\right\} \quad (15.105)$$

– *spin oscillations*

$$P_{\nu_e^L\to\nu_e^R}(t)=\{\sin(\mu_+B_\perp t)\cos(\mu_-B_\perp t)+\cos 2\theta\sin(\mu_-B_\perp t)\cos(\mu_+B_\perp t)\}^2 \\ -\sin^2 2\theta\sin(\mu_1B_\perp t)\sin(\mu_2B_\perp t)\sin^2\frac{\Delta m^2}{4p}t \quad (15.106)$$

– *spin-flavor oscillations*

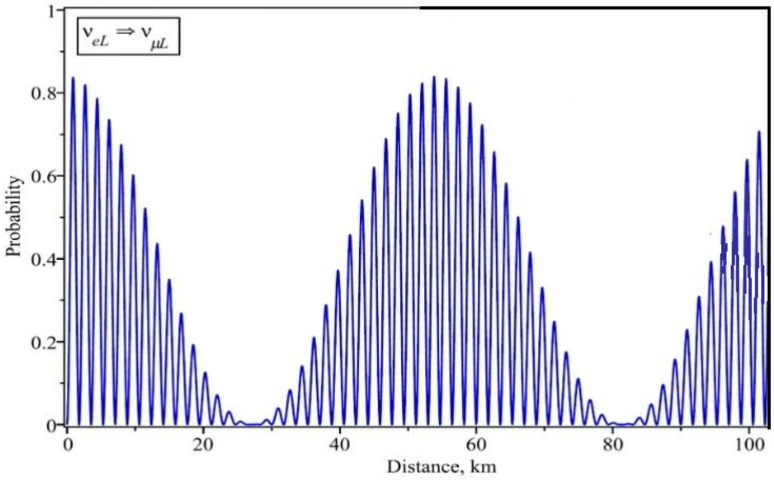
$$P_{\nu_e^L\to\nu_\mu^R}(t)=\sin^2 2\theta\left\{\cos(\mu_1B_\perp t)\cos(\mu_2B_\perp t)\sin^2\frac{\Delta m^2}{4p}t\right. \\ \left.+\sin^2(\mu_+B_\perp t)\sin^2(\mu_-B_\perp t)\right\} \quad (15.107)$$

I would like to stress that we are not accounting for the presence of matter here only the electromagnetic interaction of neutrinos with constant magnetic field. All these three phenomena are characterized by the interplay of oscillations on the vacuum frequencies $\omega_{vac}=\frac{\Delta m^2}{4p}$ and on the magnetic frequencies $\omega_B=\mu B_\perp$.

Let's consider some particular cases that more vividly indicate the beauty of these phenomena. If we suppose that the magnetic moments of the mass states of neutrino are equal to each other $\mu_1=\mu_2$ then the probability of *flavor oscillations* will be

$$P_{\nu_e^L\to\nu_\mu^L}(t) = (1-\sin^2(\mu B_\perp t))\sin^2 2\theta \sin^2\frac{\Delta m^2}{4p}t$$
$$= \left(1-P^{cust}_{\nu_e^L\to\nu_e^R}\right)P^{cust}_{\nu_e^L\to\nu_\mu^L} \qquad (15.108)$$

The amplitude of flavor oscillations on vacuum frequency $\omega_{vac}=\frac{\Delta m^2}{4p}$ is modulated by magnetic frequency $\omega_B=\mu B_\perp$.



Pic.15.8. The probability of flavor oscillations in transversal magnetic field.

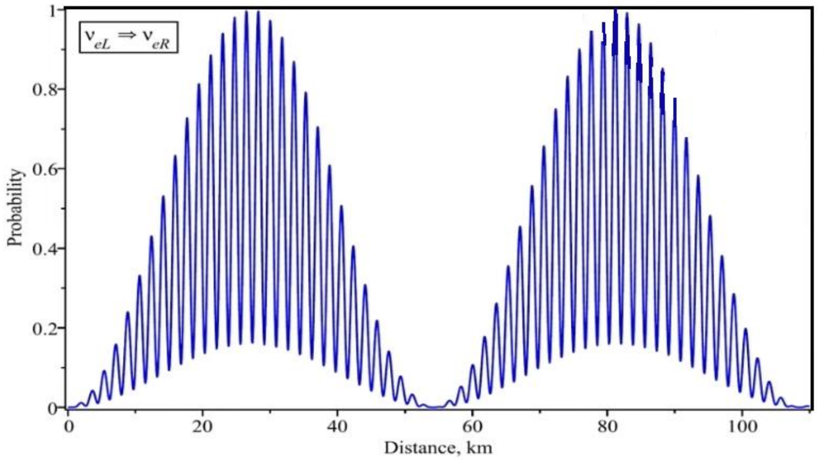
On the pic.15.8 we show that there is interplay of oscillations of on two different frequencies in the case when the transversal field is quite strong $B_\perp = 10^{16}\ G$, the neutrino energy $p = 1\ MeV$, the mass square difference $\Delta m^2 = 7\times 10^{-5}\ eV^2$ and the magnetic moments are very close to the prediction of the easiest generalization of the Standard Model $\mu_1=\mu_2=10^{-20}\ \mu_B$. It means that we choose quite realistic confirmed experimentally and from some general theoretical framework parameters. I would like to recall that if you just use the mass equal to the upper limit of $1\ eV$ (recent KATRIN experiment result is $0.8\ eV$) then the easiest generalization of the Standard Model predicts that the magnetic moment of a Dirac neutrino should be $\sim 10^{-18}\ \mu_B$, we use $10^{-20}\ \mu_B$. It means that this phenomenon rarely exist in nature, so there is no need for any new physics beyond the Standard Model only one have to accept that neutrino is a massive particle.

Again we suppose that the magnetic moments of the mass states of neutrino are equal to each other $\mu_1=\mu_2$ then the probability of *spin oscillations* will be

$$P_{\nu_e^L\to\nu_e^R}(t) = \left(1-\sin^2 2\theta\sin^2\frac{\Delta m^2}{4p}t\right)\sin^2(\mu B_\perp t)$$
$$= \left(1-P^{cust}_{\nu_e^L\to\nu_\mu^L}\right)P^{cust}_{\nu_e^L\to\nu_e^R} \qquad (15.109)$$

The amplitude of spin oscillations on magnetic frequency $\omega_B=\mu B_\perp$ is modulated by vacuum frequency $\omega_{vac}=\frac{\Delta m^2}{4p}$. Again there is interplay of oscillations of both frequencies. On the pic.15.9 the probability of spin oscillations is shown in the case when the transversal field

$B_\perp = 10^{16}\,G$, the neutrino energy $p = 1\,MeV$, the mass square difference $\Delta m^2 = 7\times 10^{-5}\,eV^2$ and the magnetic moments $\mu_1 = \mu_2 = 10^{-20}\,\mu_B$.

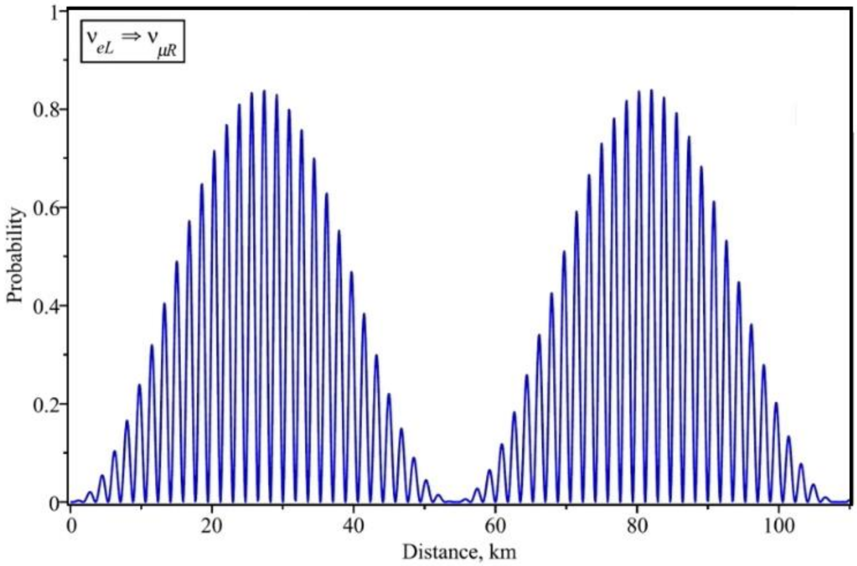


Pic.15.9. The probability of spin oscillations in transversal magnetic field.

Finally, the probability of *spin-flavor oscillations* in case of $\mu_1 = \mu_2$ then will be

$$P_{\nu_e^L\to\nu_\mu^R}(t) = \sin^2(\mu B_\perp t)\sin^2 2\theta \sin^2\frac{\Delta m^2}{4p}t$$
$$= P^{cust}_{\nu_e^L\to\nu_e^R} P^{cust}_{\nu_e^L\to\nu_\mu^L} \qquad (15.110)$$

There is interplay of oscillations on the vacuum frequencies $\omega_{vac} = \frac{\Delta m^2}{4p}$ and on the magnetic frequencies $\omega_B = \mu B_\perp$. For the same realistic choice of parameters the probability of spin-flavor oscillations is shown on pic.15.10.



Pic.15.10. The probability of spin-flavor oscillations in transversal magnetic field.

For completeness we calculated the survival probability:

$$P_{\nu_e^L\to\nu_e^L}(t) = \{\cos(\mu_+ B_\perp t)\cos(\mu_- B_\perp t) - \cos 2\theta \sin(\mu_+ B_\perp t)\sin(\mu_- B_\perp t)\}^2$$

$$- \sin^2 2\theta \cos(\mu_1 B_\perp t) \cos(\mu_2 B_\perp t) \sin^2 \frac{\Delta m^2}{4p} t \tag{15.111}$$

And just for curiosity here is a confirmation that there are no obvious mistakes in our calculations:

$$P_{\nu_e^L \to \nu_\mu^L} + P_{\nu_e^L \to \nu_e^R} + P_{\nu_e^L \to \nu_\mu^R} + P_{\nu_e^L \to \nu_e^L} = 1 \tag{15.112}$$

The discovered correspondence between flavor and spin and spin-flavor oscillations in magnetic field are sufficient for considering neutrino propagation in different astrophysical environments where magnetic fields are present.

I would like to remark about the generalization of this interplay of neutrino spin and spin-flavor oscillations in presence of moving matter. I generalize these predictions and also considered the interplay between spin, flavor and spin-flavor oscillations in case when spin oscillations are produced not by electromagnetic interaction of neutrinos with the background matter but by neutrino interaction with the transversal matter current. I just derived the similar formula for neutrino flavor oscillations as the following

$$P_{\nu_e^L \to \nu_\mu^L}^{(j_{||}+j_\perp)}(t) = \left(1 - P_{\nu_e^L \to \nu_e^R}^{(j_\perp)} - P_{\nu_e^L \to \nu_\mu^R}^{(j_\perp)}\right) P_{\nu_e^L \to \nu_\mu^L}^{(j_{||})} \tag{15.113}$$

I would like to mention that if we generalize the mixing matrix of neutrinos and also consider the additional part of this mixing matrix for Majorana neutrinos which accounts for CP-violating effects then new time type of resonances for can appear. If one also accounts for the Majorana CP-phrases it will induce new resonances as it have been indicated in our recent paper that serves as a tool for distinguishing the nature of neutrino (Dirac or Majorana).

Another very interesting study of neutrino electromagnetic properties was developed together with my colleague Konstantin Stankevich. We've been discussing the manifestation of neutrino electromagnetic properties in the effect of quantum decoherence of neutrinos. Again we incorporated the electromagnetic process of neutrino radiative decay due to neutrino coupling with the photons through nontrivial magnetic moment. This quantum decoherence can modify the oscillation pattern that might be observable in the future gigantic volume detectors such as JUNA in China, DUNE in USA and Hyper-Kamiokande in Japan.

Conclusion

I would like to point your attention on that about two three years ago there was an indication from the XENONnT experiment that was interpreted as manifestation of existing nontrivial neutrino magnetic moment on the level of the Borexino upper bound constraint. (I recall that Borexino using the fluxes of solar neutrinos obtained the upper bound for the effective magnetic moment of neutrino on the level of about $3 \times 10^{-11} \mu_B$.) But then in more recent experiments they rule out this result. So still we are waiting for more important data from the experiments.

The second stage of GEMMA experiment νGeN is taking data and it is suppose that probably in one or two years from now this experiment will reach the sensitivity to the magnetic moment about $\sim(5-9)\times 10^{-12}\mu_B$. In this case as it was shown in my paper the sensitivity to the millicharge of neutrino will be reduced to the level of about $10^{-13}e_0$. There are several new astrophysical constraints on the effective magnetic moment also in the range of $\sim 10^{-12}\mu_B$.

I would like to mention that there is the new running experiment SATURNE (Sarov Tritium Neutrino Experiment) that is realized within the program of the National Center for Physics and Mathematics in Sarov. This project is based on our paper published in Phys.Rep.D in 2019 in which we proposed to measure the coherent elastic neutrino atom scattering using the superfluid helium detector. Within this experiment we have calculated that the sensitivity to neutrino electromagnetic properties in particular magnetic moment will be on the level of $10^{-13}\mu_B$. This experiment is in preparation, we are making presentations on different conferences, last one was just few days ago on the Quark conference organized by The Institute of Nuclear Research and Joint Institute for Nuclear Research in Dubna and in Troitsk.

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